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Residual log-periodogram inference for long-run relationships

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Abstract

We assume that some consistent estimator $\widehat{\beta}$ of an equilibrium relation between non-stationary series integrated of order $d \in (0.5, 1.5)$ is used to compute residuals $\hat{u}_t = y_t - \widehat{\beta}x_t$ (or differences thereof). We propose to apply the semiparametric log-periodogram regression to the (differenced) residuals in order to estimate or test the degree of persistence δ of the equilibrium deviation u_t. Provided $\widehat{\beta}$ converges fast enough, we describe simple semiparametric conditions around zero frequency that guarantee consistent estimation of δ. At the same time limiting normality is derived, which allows to construct approximate confidence intervals to test hypotheses on δ. This requires that $d - \delta > 0.5$ for superconsistent $\widehat{\beta}$, so the residuals can be good proxies of true cointegrating errors. Our assumptions allow for stationary deviations with long memory, $0 \leqslant \delta < 0.5$, as well as for non-stationary but transitory equilibrium errors, $0.5 < \delta < 1$. In particular, if x_t contains several series we consider the joint estimation of d and δ. Wald statistics to test for parameter restrictions of the system have a limiting χ^2 distribution. We also analyse the benefits of a pooled version of the estimate. The empirical applicability of our general cointegration test is investigated by means of Monte Carlo experiments and illustrated with a study of exchange rate dynamics.

JEL classification: C14; C22

Keywords: Fractional cointegration; Semiparametric inference; Limiting normality; Long memory; Non-stationarity; Exchange rates

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1. Introduction

A substantial part of economic theory deals with long-run equilibrium relationships generated by market forces and behavioral rules. Granger (1981) and Engle and Granger (1987) were the first to formalize the idea of integrated variables sharing an equilibrium relation which turned out to be either stationary or have a lower degree of integration than the original series. They denoted this property by cointegration, signifying co-movements among trending variables which could be exploited to test for the existence of equilibrium relationships within a fully dynamic specification framework.

The presence of, at least, a unit root in economic time series is implied in many economic models as those based on the rational use of available information or on the existence of very high adjustment costs in some markets. Interesting examples include future contracts, stock prices, yield curves, exchange rates, money velocity, hysteresis theories of unemployment and, perhaps the most popular, the implications of the permanent income hypothesis for real consumption under rational expectations. Thus, most of the cointegration literature has focused on the case where variables contain a single unit root. Moreover, in most of the occasions, the equilibrium relation turned out to be modeled as a weakly stationary or short memory $I(0)$ process. Within this $I(1)/I(0)$ set up, Engle and Granger (1987) suggested a two-step estimation procedure for single equation dynamic modeling which has become very popular in applied research. First, an OLS regression is run among the levels of the series of interest. Then, Dickey–Fuller type unit root tests are performed on the residual sequence to determine whether it has a unit root. Under the null hypothesis the residuals are $I(1)$, and under the alternative the residuals are $I(0)$.

Some economic applications, however, suggest that even if the data are $I(1)$, the residual term representing the potential equilibrium error might be fractionally integrated. See, e.g., Robinson (1994a), Baillie (1996) and Gil-Alaña and Robinson (1997). Loosely speaking, a series u_t is said to be fractionally integrated of order δ, in short $I(\delta)$, if $\Delta^{\delta} u_t$ is $I(0)$, where δ is not an integer but a real number. The degree of integration determines the key dynamic or memory properties of the economic series. A fractionally integrated process is stationary if $\delta < 0.5$ and nonstationary otherwise (cf. Granger and Joyeux, 1980; Hosking, 1981). In spite of being nonstationary, if $0.5 \leqslant \delta < 1$ the process is mean-reverting with transitory memory, i.e. any random shock has only a temporary influence on the series, in contrast with the case when $\delta \geqslant 1$, where the process is both nonstationary and not mean-reverting with permanent memory, i.e., any random shock having now a permanent effect on the future path of the series. Consequently, a wide range of dynamic behavior is ruled out a priori if δ is restricted to integer values and a much broader range of cointegration possibilities is permitted when fractional cases are considered. More importantly, now the degree of memory of the residual series, δ, is a parameter suitable, in principle, of estimation and testing by means of any of the existing methods.