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Option-implied risk aversion estimation

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In this paper, following the Jackwerth (2000) work, we estimate the risk aversion function on the French market implied by the joint observation of the cross-section of option prices and time-series of underlying asset returns from a high-frequency CAC 40 index options. We recover risk aversion empirically around the 2007 crisis using two different Risk-Neutral densitiy estimation approaches and for different options maturities. We studied simultaneously the impacts of the time-to-maturity, the risk-neutral density estimation model and the time periods of the study on the risk aversion function. Our findings show that the estimated risk-aversion functions that we compute from both mixture of log–normals and jump diffusion models for different maturities and three time periods are unfortunately not positive and monotonically downwards sloping as suggested by the standard assumptions of the economic theory. We note also that the values of the risk aversion are very close to that reported by studies based on consumption data. Moreover, for the three chosen trading days, if we change the time-to-maturity, we note that the risk aversion function shape is not affected for both one and two months maturities which corroborates the findings of Jackwerth (2000) but for the three months maturity, the risk aversion function shape is clearly affected and the U-shaped pattern is more pronounced for the longer maturities. Furthermore, we find that whatever the chosen trading date is, if we have the same time-to-maturity, we keep the same shape of the curve as well as the risk aversion function shape is not substantially affected if we extract the RND from a jump diffusion model or a mixture of log–normals model mainly for the short time-to-maturities. Compared to the time-to-maturity impact on the implied risk aversion, the selected RND model impact is less significant. A noteworthy finding is that the variation interval of the implied risk aversion on the postcrisis, when the market trend is upward, is more remarkable and large than the precrisis and postcrisis for the long-run maturity. Concluding, the risk aversion response is asymmetric depending on the Risk-Neutral density, time to maturity options and the period of study.

1. Introduction

The eighties have marked the beginning of the academic research regarding the application of risk aversion levels to areas of risk management and quantitative finance. Among these studies, we find the Blume and Friend (1975) work that attempts to identify the nature of the utility function of households from an analysis of the Heritage composition of 2100 of

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them. Their results led them to conclude that the assumption of the constant relative risk aversion is an acceptable approximation of the reality. Similarly, Szpiro (1986) verifies that the assumption of constant relative risk aversion is true and finds that the coefficient of risk aversion is between about 1.2 and 1.8. In the same context, Nakamura (2007), tests the stability of the risk aversion coefficient to Japanese daily data between 1973 and 1991 and shows that it is invariant. Furthermore, Nishiyama (2007) studied the effect of a change in the coefficient of risk aversion of U.S. banks versus Japanese banks. He noted that the increase in risk aversion of U.S. banks is unambiguously associated with the Asian crisis, while the increase in Japanese banks risk aversion is only weakly associated.

Alonso, Ganzala, and Tusell (1990) estimate the relative risk aversion coefficient for the Spanish Stock Market between 1965 and 1984. As Rosenberg and Engle (1997) and Aït-Sahalia and Lo Anderw (2000) use the relationship between the Risk-Neutral and the subjective densities and the risk aversion function to derive the marginal utility function, Coutant (1999) and Enzo, Handel, and Härdle (2006) showed that the risk aversion function varies over time. Jackwerth (1996, 2000) empirically derive risk aversion functions implied by S&P500 index options prices around the 1987 crash. He find that precrash risk aversion functions are positive and decreasing by wealth but postcrash ones are partially negative and partially increasing and irreconcilable with standard assumptions made in economic theory. Other recent research including that of Pérignon. and Villa. (2002). These authors apply the same technique of Aït-Sahlia and Lo (2000) in order to estimate the risk aversion function using the CAC 40 index options. They have defined a new formula for the risk aversion function which is "the geometric risk aversion measure". Finally, we find the study of Bliss and Panigirtzoglou (2004) which covers the estimation of the risk aversion function for different maturities. They use two utilities functions: Exponential and Power and derive the risk aversion coefficient from the S&P 500 and FTSE 100 indexes options.

In this paper, following the Jackwerth (2000) work, we estimate the risk aversion function on the French market implied by the joint observation of the cross-section of option prices and time-series of underlying asset returns. Our paper is the first dealing with European options to recover risk aversion empirically around the 2007 crisis, i.e. dividing the period of study into three sub-periods: precrisis, crisis and postcrisis sub-periods, using two different Risk-Neural densitiy estimation approaches and for different options maturities. This paper is the first that studies simultaneously the impacts of the time-to-maturity, the risk-neutral density estimation model and the time periods of the study on the risk aversion function, that is the study of the asymmetric response of risk aversion.

The paper is organized as follows. Section 2 presents the fundamental relationship that exists between aggregate risk-neutral and subjective probability distributions and risk aversion functions. Section 3 describes our data and procedures used for the empirical work: the implied risk-neutral density estimation methods and subjective density estimation approach. Section 4 presents and analyses the empirical results. Section 5 concludes.

2. 2 Implied risk aversion functions

Risk aversion allows not only to understand the agent's risk behaviour but also to specify the shape of the utility function. Two measures of risk aversion have been proposed in the literature: absolute risk aversion (A_a), which is defined in presence of exogenous risks, and relative risk aversion (A_r), defined in presence of endogenous or proportional risks. Pratt (1964) and Arrow (1971) define two measures:

$$A_a = -U''(W)/U'(W) \tag{1}$$

$$A_r = -WU''(W)/U'(W) = A_a W \tag{2}$$

where U is the utility function and W is the individual wealth. Arrow (1971) hypothesizes that most investors display decreasing absolute risk-aversion (DARA) and increasing relative risk-aversion (IRRA) with respect to wealth. He points out that the DARA "seems supported by everyday observations"[1] but he establishes that IRRA "was not easily confortable with intuitive evidence".[2] Theoretically, we can easily see the dependence of A_a and A_r on W.

$$A_r = A_a W \tag{3}$$

$$\frac{dA_r}{dW} = A_r + \frac{dA_a}{dW} \tag{4}$$

Therefore, if dA_a/dW is negative (DARA), dA_r/dW can be positive, negative or null since $A_a$and W are positive. Besides, usual utility functions impose implicitly decreasing (D), constant (C) or increasing (I) absolute risk aversion (ARA) and relative risk aversion (RRA). For instance, the logarithmic utility function is DARA and CRRA, the power utility function DARA and CRRA and the negative exponential utility function CARA and IRRA.

Blume and Friend (1975) specifies that A_r is constant with changing wealth. Lucas model (1978) provides an accurate framework to estimate empirically the relative risk aversion coefficient. The dynamic optimization problems of economic agents typically imply a set of stochastic Euler equations that must be satisfied in equilibrium. These Euler equations imply a

[1] See Arrow (1971), p. 96.
[2] See Arrow (1971), p. 97.