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Stiffness design of heterogeneous periodic beam by topology optimization with integration of commercial software

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ABSTRACT

A topology optimization method is developed for microstructure design of heterogeneous periodic beam structure aiming at its extreme or specified effective stiffness. The effective stiffness is calculated using a FEM formulation of asymptotic homogenization method for heterogeneous periodic beam. Sensitivity of stiffness to the density design variable is derived analytically based on the solution of unit cell problems under corresponding generalized strain fields. Implementation of optimization procedure is generalized to take full advantage of commercial FEM software capabilities, with several examples presented to demonstrate its effectiveness. It is shown here the proposed method is extendible to periodic truss beam design.

1. Introduction

Recent development of manufacturing technology make it possible to realize many different variety of engineering structure configurations to meet the requirement of multifunctional, lightweight, and efficient performance. One important category is heterogeneous periodic continuum: structure consisting of unit cells arranged periodically in one, two or three dimensions (Fig. 1). Among them, slender periodic heterogeneous beams such as sandwich beams, ribbed boxes and stranded ropes are widely applied in architectural, aeronautical and automotive industries. Structural analysis and optimum design of slender periodic heterogeneous beams become important research topics. For example, several recent research papers address structural analysis and optimization of composite corrugated skins, metallic corrugated core sandwich panels, beams with periodically variable cross-sections and graded corrugated truss core composite sandwich beams [1–4].

Periodic continuum with unit cell arranged in two or three dimensions behaves generally as a solid elastic continuum with effective material properties. The configuration and composition of their unit cell microstructure determine the overall performance of the macro structure, and the microstructure design of the unit cell is also known as material design. Sigmund [5] introduced the inverse homogenization method into material design. Combining 2D or 3D AH (Asymptotic Homogenization) theory, extreme or unique material properties can be obtained through topology optimization of the unit cell, e.g. material with negative Poisson's ratio [5]. Many researches followed subsequently with the objective function of mechanical or multifunctional performance. For example, Guest et al. [6,7] and Xu and Cheng [8] obtained material microstructure considering both stiffness and isotropic permeability coefficient by the inverse homogenization method. Torquato et al. [9] combined the thermal and electrical conductivities with equal weight for the material design problem. However, all the above work assumes that the continuum has periodicity in three dimensions or within certain plane, where well developed AH (Asymptotic homogenization) method applies.

There were several earlier investigations on the topology optimization of beam structures. Kim and Kim [10] formulated a section topology optimization technique to find the optimal cross section configuration with the objective function of a weighted sum of bending and torsional rigidities. Liu and An [11] established a topology optimization formulation based on an anisotropic beam theory considering warping of sections and coupling among deformations. Blasques and Stolpe [12] presented a novel framework for simultaneous optimization of topology and laminate properties in structural design of laminated composite beam cross sections. However, the above beam optimizations are limited to the cross section design, which assumes that the cross section along the beam axis keeps constant. If cross section of the heterogeneous beam structure varies periodically in its axial direction, and its out-axial effective properties is to be optimized, several difficulties need to be addressed in a general method.

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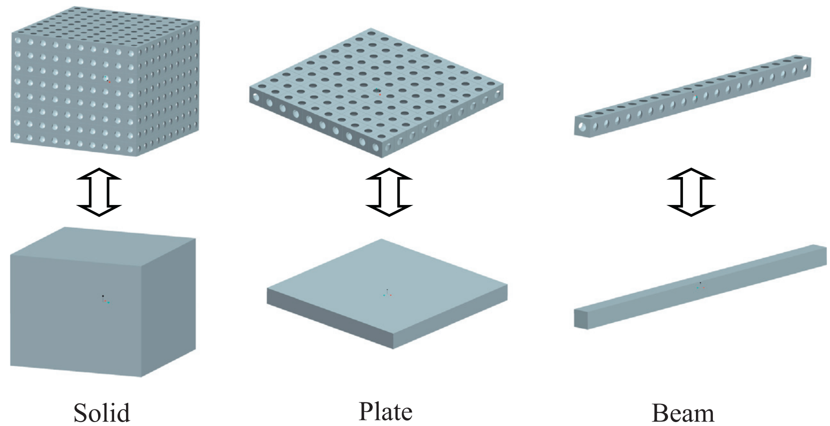

Fig. 1. Various periodic structures and their homogeneous models.

The first difficulty is model reduction and effective stiffness prediction of periodic heterogeneous, beam-like structure. The cross sectional dimensions of periodic heterogeneous beam are significantly smaller than their length along the axial direction, the conventional numerical methods to analyze and optimize the overall behavior of these structures lead to heavy computations. To reduce the computational effort, this kind of periodic beam structures needs to be simplified as Euler–Bernoulii beam or Timoshenko beam by dimension reduction in addition to homogenization.

Representative volume element (RVE) method and asymptotic homogenization (AH) method are two popular methods to predict effective properties of periodic structures. The RVE method has clear physical or mechanical meaning but no rigorous mathematical foundation, so it provides approximate solutions and is easy to implement. The AH method has rigorous mathematical foundation and has been successfully used in predicting effective modulus of 3D and in-plane effective modulus of 2D periodic materials both analytically and numerically [13–15]. Moreover, the sensitivity information can be obtained from the AH method, which is the foundation of the gradient-based optimization algorithm. However, FEM implementation of AH for periodic beam and plate, which does not have periodicity in its cross section or its thickness direction, has been a challenging subject.

Cheng et al. [16] developed a New Implementation of the Asymptotic Homogenization (NIAH) method to predict effective properties of periodic materials with periodicity in 3D or 2D in plane, and this new FEM implementation has been extended to the homogenization method for periodic heterogeneous plate and shell structures by Cai et al. [17]. The new implementation has a rigorous mathematical foundation of the AH method, and can be simply implemented in commercial software as a black box. Based on the AH theory of heterogeneous periodic beams developed by Kolpakov and Kalamkarov [18–20], Yi et al. [21] established a FEM formulation for the AH theory of periodic heterogeneous beam structures in the same framework of the NIAH method. With this FEM implementation of AH method of periodic beam structures, one can obtain the effective stiffness of beam with complicated microstructure in its 3D unit cell with reduced computational cost.

The topology optimization of the unit cell microstructure is another challenge because structural analysis of three dimensional elasticity problem is computationally intensive. Structural optimization algorithms are generally classified into gradient-based algorithm and population-based algorithm [22–24]. The population-based algorithm simulates the natural phenomenon through random processes to search the global optimal solution in the design space and needs large number of function evaluations. The gradient-based algorithm searches the optimal solution in special gradient direction, thus it has high efficiency with fewer iterations and faster convergence. For topology optimization of microstructure design with thousands or even more design variables, each iteration requires three dimensional FEM analysis, thus the gradient-based algorithm is more suitable for microstructure design problems.

For gradient-based optimization, the availability of analytical sensitivity analysis has significant influence to the optimization efficiency. The sensitivity-based optimization algorithm can actually take longer than the zero-order optimization algorithm if the sensitivity calculation is inefficient, for example, with the finite difference method. Herein we show with the NIAH implementation recently proposed by authors, analytical sensitivity analysis is possible to enable gradient-based optimization algorithm for periodic heterogeneous beam design.

Such topology optimization can be even more efficient by proper integration with commercial FEM software such as ANSYS, ABAQUS, to leverage their pre-processing and post-processing ability, expanded element types and efficient linear or eigenvalue matrix solver. To integrate the newly developed structural optimization algorithm with the existing commercial software, one key problem is sensitivity calculation, which most commercial software lack. Such demand drove the development of semi-analytic sensitivity method [25] and its many extensions. Zhang et al. [26] developed a practical tool to deal with sizing sensitivity analysis with ABAQUS. Jeong et al. [27] proposed a novel approach using indirect calculation of internal finite element information for a stress-based topology optimization problem. Henrichsen et al. [28] performed optimum stiffness design of laminated composite structures using the commercially available programs ANSYS and MATLAB.

This paper develops a topology optimization method for the microstructure design of periodic beam structure under the available material volume. The FEM element density of the unit cell is the topology design variable. The objective is to design extreme or specified effective stiffness using NIAH method efficiently. Our analytically derived sensitivities of the effective stiffness is proportional to the element strain energy or mutual strain energy corresponding to the generalized unit strain fields, extracted using commercial software and indirect calculation of internal finite element information. Section 2 focuses on the mathematical formulation of microstructure design problem for periodic heterogeneous beam structure. In section 3, we introduce the FEM implementation of asymptotic homogenization theory for periodic beam structure. Section 4 describes in detail sensitivity analysis and the integration of evaluation of the objective function, sensitivity