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High-fidelity prediction of crack formation in 2D and 3D pullout tests

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ABSTRACT

This paper presents the 2D and 3D numerical analysis of pullout tests on steel anchorages in concrete blocks using standard and mixed finite elements. A novel (stabilized) mixed formulation in the variables of total strain ε and displacements u is introduced to overcome the intrinsic deficiencies of the standard displacement-based one in the context of localization of strains, such as mesh dependency. The quasi-brittle behavior of concrete is described through an elastoplastic constitutive law with a local Rankine yielding criterion. The proposed formulation is shown to be a reliable and accurate tool, sensitive to the physical parameters of the pullout tests, but objective with respect to the adopted FE mesh. Furthermore, the mixed ε/u finite element is able to capture the correct failure mechanism with relatively coarse discretizations. At the same time, the spurious behavior of the standard formulation is not alleviated by mesh-refinement.

1. Introduction

Concrete is widely used in the context of civil structures, and yet, it is a rather complex material. Having an inherent internal heterogeneity and reacting to environmental conditions makes the post-peak non-linear behavior and the subsequent failure very difficult to predict, if not impossible. Then, it is clear the key role of experimental tests in the context of concrete structures reliability. Within the extensive literature in the field, one can recall L-shaped panels tests by Winkler et al. [1], wedge splitting by Trunk [2], single edge notched beams by Guinea et al. [3], Gálvez et al. [4], mixed mode fracture tests by Nooru-Mohamed [5], Ballatore et al. [6] and indentation tests by Berenbaum and Brodie [7].

In this work, the pullout test is addressed, being one of the most interesting experimental techniques to evaluate the strength of concrete and the overall behavior of embedded anchorages. First accounts of the pullout test come from Abrams [8] and Slater et al. [9]. Later, Elfgren et al. [10] and Peier [11] introduced numerical modeling as a validation of the experimental procedure. While the laboratory setup can be easily reproduced, the test outcomes are strictly dependent on the choice of various parameters as the mechanical properties of the employed materials, the size of specimens and the anchorage embed depth. The influential works of Eligehausen and Sawade [12], Bažant et al. [13], Ožbolt et al. [14], Karihaloo [15] emphasized the structural size effect on the pullout test and its influence on the energy dissipated in the cracking process. Most recent advances are related to the possibility of testing the coupling between FRP composites and concrete [16,17]. Hereafter, the attention will be focused on the reported experiments from Dejori [18], Thenier and Hofstetter [19] for the 2D case, whereas the work by Gasser and Holzapfel [20], Areias and Belytschko [21] will be considered for the 3D case.

The methodology used to assess physical properties of specimens during a test is as critical as the details that characterize the particular experiment. Slight changes in the application of a load or in the supporting system can affect severely the results without a clear explanation. Permutating over multiple experimental settings can help to understand better the numerous variables involved, but, in reality, not all combinations are possible, due to limitations in controlling the test bench as well as time and cost restrictions. Hence, it is in this framework that numerical simulations play a fundamental role for the prediction and the possible improvement of experiments.

Recently, the authors presented a general purpose finite element technology for compressible and incompressible plasticity [22], which has successfully tackled geotechnical problems [23]. The proposed mixed strain–displacement (ε/u) formulation has been applied to local constitutive models in plasticity, in the framework of the smeared crack approach [24]. In problems involving strain-localization, standard finite elements present numerous limitations, being affected by spurious mesh-biased dependence and stress locking. In such cases, the sensibility required to evaluate the change of results with respect to diverse boundary conditions can be overshadowed by the lack of precision in the inelastic range of classical displacement-based finite

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elements. On the contrary, the mixed ε/u finite element formulation is capable to overcome these issues, predicting effectively the peak load, the failure mechanism and the localization bands. The method is also free from any mesh dependence and it does not require any additional tracking technique. This is a substantial clinching feat, as, leaving out theoretical qualms and from the factual point of view, local crack tracking procedures are very difficult to implement in 3D and global methods cannot deal with crack branching.

Taking advantage of its reliability, the proposed method is applied in this work to the pullout problem, using 2D and 3D elements with linear interpolations of both the displacement and total strain fields. In previous works [25–27], the formulation has been used in the context of isotropic damage models. In this work, though, a plasticity model based on the Rankine failure criterion is used, similarly to those used in Refs. [20,28–31]. Furthermore, plasticity with strain softening has been proved to be able to reproduce structural size-effect in a wide range of scales and, particularly, in engineering-size problems [25].

The objective of the paper is proving that the use of an appropriate finite element technology is crucial for the study of the experimental setting and for the assessment of the results, even in the case of a very well known application, as the pullout test is. The outline of the paper is as follows. First, the displacement-based and the mixed strain–displacement formulations are introduced. Then, the plasticity constitutive model is presented and the Rankine yielding criterion is extended in the case of multi-axial loading condition. A regularization of the singular points, useful to avoid indetermination issues in the return mapping algorithm, is shown. Finally, numerical simulations of 2D and 3D pullout tests are presented: standard and mixed finite element analyses are compared, demonstrating both the sensibility to changes on the boundary conditions and the replication of experiments. The results show that the mixed ε/u finite element provides reliable and high quality outcomes when compared to the standard irreducible formulation.

2. Governing equations

A solid body $\mathscr{B}$ occupying the space domain Ω is described by the position X of each point with respect to a system of coordinates x, y, z.

On the one hand, every point of such domain has a displacement $\boldsymbol{u}$ and a total strain $\boldsymbol{\varepsilon}$. Both displacements and strains are considered small. The compatibility condition relates both fields as:

$$-\boldsymbol{\varepsilon} + \boldsymbol{\nabla}^s \boldsymbol{u} = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{\nabla}^s(\cdot)$ is used to denote the symmetric gradient operator. On the other hand, the equilibrium of forces in (quasi-)static conditions states that:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \boldsymbol{f} = \boldsymbol{0} \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\boldsymbol{f}$ are the external forces applied to the body. The symbol $\boldsymbol{\nabla} \cdot (\cdot)$ refers to the divergence operator. The total strain $\boldsymbol{\varepsilon}$ is decomposed additively in the elastic $\boldsymbol{\varepsilon}_e$ and the plastic $\boldsymbol{\varepsilon}_p$ parts. The link between Cauchy's stress and the total strain is given by the constitutive law:

$$\boldsymbol{\sigma} = \mathbb{C} : \boldsymbol{\varepsilon}_e = \mathbb{C} : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p\right) \tag{3}$$

where $\mathbb{C}$ is the fourth order elastic constitutive tensor. Recalling Eqs. (1) and (2), the problem reads:

$$\begin{aligned} &-\boldsymbol{\varepsilon} + \boldsymbol{\nabla}^s \boldsymbol{u} = \boldsymbol{0} \\ &\boldsymbol{\nabla} \cdot \left[\mathbb{C} : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p\right)\right] + \boldsymbol{f} = \boldsymbol{0} \end{aligned} \tag{4}$$

This set of equations represents the strong form for the mixed problem involving the unknown fields of displacements $\boldsymbol{u}$ and total strains $\boldsymbol{\varepsilon}$ in the case of plasticity. In order to obtain a symmetric system, the first equation is pre-multiplied by the elastic constitutive tensor $\mathbb{C}$:

$$\begin{aligned} &-\mathbb{C} : \boldsymbol{\varepsilon} + \mathbb{C} : \boldsymbol{\nabla}^s \boldsymbol{u} = \boldsymbol{0} \\ &\boldsymbol{\nabla} \cdot \left[\mathbb{C} : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p\right)\right] + \boldsymbol{f} = \boldsymbol{0} \end{aligned} \tag{5}$$

The irreducible problem, in terms of the displacement field $\boldsymbol{u}$ only, is recovered substituting the first equation into the second, to yield:

$$\boldsymbol{\nabla} \cdot \left[\mathbb{C} : \left(\boldsymbol{\nabla}^s \boldsymbol{u} - \boldsymbol{\varepsilon}_p\right)\right] + \boldsymbol{f} = \boldsymbol{0} \tag{6}$$

With proper conditions on the boundary $\partial\Omega$ and evolution laws for the plastic strain field [32], both irreducible and mixed formulations provide a well posed boundary value problem.

3. Irreducible finite elements

Recalling the strong form in Eq. (6), the corresponding weak problem can be written as:

$$\int_\Omega \boldsymbol{v} \cdot \left(\boldsymbol{\nabla} \cdot \left[\mathbb{C} : \left(\boldsymbol{\nabla}^s \boldsymbol{u} - \boldsymbol{\varepsilon}_p\right)\right]\right) + \int_\Omega \boldsymbol{v} \cdot \boldsymbol{f} = 0 \quad \forall \boldsymbol{v} \in \mathbb{V} \tag{7}$$

where $\mathbb{V}$ is the space of test functions which are square integrable. Integrating by parts, the forcing terms can be extracted as:

$$\int_\Omega \boldsymbol{\nabla}^s \boldsymbol{v} : \mathbb{C} : \left(\boldsymbol{\nabla}^s \boldsymbol{u} - \boldsymbol{\varepsilon}_p\right) = F(\boldsymbol{v}) \tag{8}$$

where the boundary terms accounting for body forces $\boldsymbol{f}$ on Ω and tractions $\boldsymbol{t}$ on the boundary $\partial\Omega$ are collected in the term

$$F(\boldsymbol{v}) = \int_\Omega \boldsymbol{v} \cdot \boldsymbol{f} + \int_{\partial\Omega} \boldsymbol{v} \cdot \boldsymbol{t} \tag{9}$$

The discretized version of Eq. (8) is obtained by selecting a finite set of interpolation functions for the displacement field as well as the test function as:

$$\boldsymbol{u} \to \boldsymbol{u}_h = \sum_{i=1}^{n_{pts}} \boldsymbol{v}_h^{(i)} \boldsymbol{u}_h^{(i)} \quad \boldsymbol{v}_h \in \mathbb{V}_h \tag{10}$$

such that the discrete functional space $\mathbb{V}_h$ is a subset of the continuous version $\mathbb{V} \subseteq H^1(\Omega)^{dim}$. From Eq. (8), the final discrete system of equations reads:

$$\int_\Omega \boldsymbol{\nabla}^s \boldsymbol{v}_h : \mathbb{C} : \left(\boldsymbol{\nabla}^s \boldsymbol{u}_h - \boldsymbol{\varepsilon}_p\right) = F(\boldsymbol{v}_h) \tag{11}$$

For the standard finite element interpolation, linear triangles P1 and quadrilateral Q1 are considered in this work.

4. Mixed ε–u finite elements

4.1. Galerkin method

The weak form of the set of equations in (5) is:

$$\begin{aligned} &-\int_\Omega \boldsymbol{\gamma} : \mathbb{C} : \boldsymbol{\varepsilon} + \int_\Omega \boldsymbol{\gamma} : \mathbb{C} : \boldsymbol{\nabla}^s \boldsymbol{u} = 0 \quad \forall \boldsymbol{\gamma} \in \mathbb{G} \\ &\int_\Omega \boldsymbol{v} \cdot (\boldsymbol{\nabla} \cdot \boldsymbol{\sigma}) + \int_\Omega \boldsymbol{v} \cdot \boldsymbol{f} = 0 \quad \forall \boldsymbol{v} \in \mathbb{V} \end{aligned} \tag{12}$$

In this case, besides the functional space $\mathbb{V}$ for the test functions $\boldsymbol{v}$ of the displacement field $\boldsymbol{u}$, it is required to introduce the set of test function tensors for the strain $\boldsymbol{\varepsilon}$ pertaining to $\mathbb{G}$. Integrating by parts the second equation, it can be written: