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Alleviation of parasitic slip in finite element analysis of composite beams

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ABSTRACT

In the conventional displacement-based finite element analysis of composite beams that consist of two Euler–Bernoulli beams juxtaposed with a deformable shear connection, the coupling of the transverse and longitudinal displacement fields may cause oscillations in interlayer slip field and reduction in optimal convergence rate, known as slip locking. This locking phenomenon is typical of multi-field problems of this type, and is known to produce erroneous results for the displacement based finite element analysis of composite beams based on cubic transverse and linear longitudinal interpolation fields. In this study, a very simple and novel procedure is introduced to eliminate the parasitic slip in the finite element analysis of composite beams. A systematic solution of the differential equations of equilibrium is also provided, and an exact element is developed in the paper. Numerical results presented illustrate the accuracy gained based on the proposed modification to the basic finite element formulation. Solutions based on the exact element provide benchmark results for the performance of the proposed formulation.

1. Introduction

Composite beams that consist of two or more different components juxtaposed with a shear connection find widespread applications, especially in the floor system and bridge design. Examples may include steel–concrete composite beams where concrete provides the compressive strength, fire resistance, and floor surface, while the steel possesses high tensile strength and has the advantage of rapid erection, and the layered wood systems commonly used in housing for floors, walls and roof structures, where composite action provides a stiffer and stronger structural system. In many civil engineering applications involving steel–concrete beams connected with shear studs or layered wood systems connected with nails, the connection is not infinitely rigid, thus permitting relative slip between the components while preserving the contact. The interlayer slip between the two components significantly affects the behavior of the composite beam and a mathematical model that considers the interlayer slip was initially introduced by Newmark et al. [1], in which two Euler–Bernoulli beams are connected by assuming that vertical separation does not occur between the components. There is a vast literature on the improvement of models for composite beam analysis, e.g. [2–6]. The scope of the current study is limited to Newmark's model [1], which is commonly adopted in structural engineering applications. Analytical solution methods based on Newmark's model can be found in Girhammar and Gopu [7], Faella et al. [8] and Girhammar and Pan [9]. Displacement-based beam-column type finite element formulations were developed by Arizumi et al. [10], and Daniels and Crisinel [11]. However, for stiff interlayer connections, displacement-based finite element formulations may suffer from the so-called slip-locking phenomenon, which was initially investigated by Dall'Asta and Zona [12,13]. The consistent interpolation strategy was adopted for composite beam analysis by Dall'Asta and Zona [12,13] to develop locking-free displacement-based finite element formulations in which additional internal nodes have to be introduced to match the interpolation functions of the axial and transverse displacement fields.

Assumed strain formulations and the kinematic interpolation strategy that alleviate slip-locking behavior for stiff connections were introduced by Erkmen and Bradford [14,15]. A meshfree formulation based on the matching field strategy that completely eliminates slip-locking was recently developed in [16]. A convenient practice in the modeling of composite beams for the cases with infinitely rigid connections is to connect the two conventional beam type finite element components by using a rigid bar to connect the end nodes of the two components or use master–slave type kinematic constraints to express the nodal degrees-of-freedom of one of the members in terms of the other. Eccentricity related numerical issues in that case as reported by Gupta and Ma [17] are corrected by using fictitious members and springs in Erkmen et al. [18,19]

The novel idea in this paper is to use consistent interpolation for the slip field which introduces additional parameters, however eliminate those parameters by imposing an explicit constraint condition. As a result of this procedure, slip oscillations do not occur and thus the element is suitable to be used for the analysis

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of composite beams with stiff connections. The idea of imposing explicit constraints has been used for shear deformable beam and plate theories in the past, e.g. [20–23]. The mechanism for shear locking in Timoshenko beams and slip locking in composite beams is similar as discussed in [14]. The element requires no additional internal nodes or static condensation procedure and yet, completely eliminates parasitic slip field. A systematic solution of the differential equations of equilibrium is also provided, and an exact element is developed in the paper. Solutions based on the exact element provide benchmark results for the performance of the proposed formulation. Examples are presented to illustrate the performance and the numerical characteristics of the simple and novel finite element developed herein.

2. Composite beam-column analysis

2.1. Kinematic model

The composite beam-column is composed of a top and a bottom Euler–Bernoulli beam which are referred to as layers 1 and 2. The composite cross-section is thus represented as $A = A_1 + A_2$, where A_1 and A_2 are the cross-sections of layers 1 and 2, respectively as shown in Fig. 1(a).

2.2. Strains

According to Newmark's model, the strain diagram is determined uniquely by the curvature of the vertical deflection v'' with respect to an arbitrary reference axis and the derivatives of the longitudinal displacements at the centroid of each layer w_1' and w_2' as shown in Fig. 1. Slip displacement between the two layers Γ can be obtained in terms of the slope of the vertical deflection v' and the longitudinal displacements at the centroids of the layers w_1 and w_2.

$$\varepsilon_1 = w_1' - (y - h_1)v'', \tag{1}$$

$$\varepsilon_2 = w_2' - (y + h_2)v'', \tag{2}$$

$$\Gamma = w_2 - w_1 + hv', \tag{3}$$

where prime (′) denotes the derivative with respect to longitudinal coordinate z.

3. Basic finite element formulation

3.1. Displacement based finite element formulation

A displacement based finite element formulation can be developed by employing the total potential energy functional, i.e.

$$\Pi = \frac{1}{2}\int_L \int_{A_1} E_1 \varepsilon_1^2 \mathrm{d}A\mathrm{d}z + \frac{1}{2}\int_L \int_{A_2} E_2 \varepsilon_2^2 \mathrm{d}A\mathrm{d} + \frac{1}{2}\int_L \int_b \rho \Gamma^2 \mathrm{d}x\mathrm{d}z - \Pi_{ext}, \tag{4}$$

where the first and second integrals are the bending energies of the two layers, the third integral is due to the elastic deformations of the shear connection in which ρ is the stiffness of the shear connection (force/length^3) which is the shear stresses in longitudinal direction for unit slip, b is the width of the effective intersection surface between the two layers and Π_{ext} is the work done by external forces. In a displacement-based finite element formulation the longitudinal displacement fields w_1, w_2 and the derivative of the vertical displacement field v' can be expressed in terms of the selected interpolation functions as

$$\begin{Bmatrix} w_1 \\ w_2 \\ v' \end{Bmatrix} = \begin{bmatrix} \mathrm{M}(z) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{M}(z) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathrm{N}'(z) \end{bmatrix} \begin{Bmatrix} \mathbf{w}_{\mathrm{N}1} \\ \mathbf{w}_{\mathrm{N}2} \\ \mathbf{v}_{\mathrm{N}} \end{Bmatrix} = \mathbf{B}(z)\mathbf{U}, \tag{5}$$

where M(z) and N(z) are the vectors of interpolation functions for the longitudinal and the vertical displacement fields, respectively. In Eq. (5), $\mathbf{B}(z)$ is the discrete slip matrix and $\mathbf{U}$ is the vector of nodal displacements composed of the vectors of nodal longitudinal displacements at the centroids of both layers $\mathbf{w}_{\mathrm{N}1}$, $\mathbf{w}_{\mathrm{N}2}$ and the vertical displacement $\mathbf{v}_{\mathrm{N}}$, i.e.

$$\mathbf{U}^{\mathrm{T}} = \langle \mathbf{w}_{\mathrm{N}1}^{\mathrm{T}} \quad \mathbf{w}_{\mathrm{N}2}^{\mathrm{T}} \quad \mathbf{v}_{\mathrm{N}}^{\mathrm{T}} \rangle. \tag{6}$$

By substituting Eqs. 1, 2, 3, (5) into Eq. (4), the total potential energy functional can be written as

$$\Pi = \frac{1}{2}\mathbf{U}^{\mathrm{T}} \int_L \mathbf{B}_{\mathrm{d}}^{\mathrm{T}}(z)\mathbf{D}\mathbf{B}_{\mathrm{d}}(z)\mathrm{d}z\mathbf{U} + \frac{1}{2}\mathbf{U}^{\mathrm{T}} \int_L \mathbf{B}^{\mathrm{T}}(z)\mathbf{D}_{\rho}\mathbf{B}(z)\mathrm{d}z\mathbf{U} - \mathbf{F}\mathbf{U}, \tag{7}$$

in which $\mathbf{B_d}(z) = \mathbf{B}'(z)$, $\mathbf{F}$ is the energy equivalent nodal external load vector,

$$\mathbf{D} = \begin{bmatrix} E_1A_1 & 0 & 0 \\ 0 & E_2A_2 & 0 \\ 0 & 0 & E_1I_1 + E_2I_2 \end{bmatrix}, \tag{8}$$

and

$$\mathbf{D}_{\rho} = \rho b \begin{bmatrix} 1 & -1 & -h \\ -1 & 1 & h \\ -h & h & h^2 \end{bmatrix}, \tag{9}$$

where I_1 and I_2 are the moments of inertia of the layers with respect to their horizontal principal axes passing through the centroids of each cross-section and h is the distance between these centroids

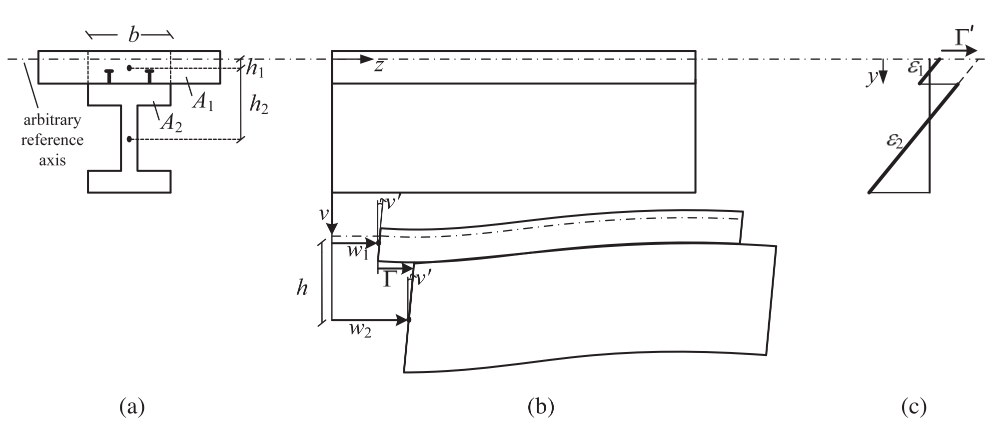

Fig. 1. Composite beam; (a) cross-section, (b) displacements, (c) strains.