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Adaptive topology optimization with independent error control for separated displacement and density fields

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ABSTRACT

This paper proposes a new adaptive method for topology optimization of structures, by using independent error control for the separated displacement and material density fields. Since the arrangement of the density points is unnecessarily associated with the analysis mesh in the topology optimization based on analysis-separated density interpolation, the refinements of each field can be separately implemented. Here, the analysis mesh is refined to improve the computational accuracy of the displacement field and the associated strain field within certain local regions (e.g. the regions around concentrated loading points and displacement restrictions), while the density field is refined in the regions between fully solid and void phases to improve the geometrical description quality of design boundaries. With such a strategy, the refinements of the analysis mesh and the density field are naturally separated and not bond together anymore. Actually, each refinement process is independently performed only when and where necessary. Numerical examples show that the proposed method can achieve high-quality and high-accuracy optimal solutions comparable to those obtained with fixed globally fine analysis meshes and fine distributed density points, but with much less computational cost.

1. Introduction

Topology optimization is a powerful tool for seeking the best structural layouts in the conceptual design stage [1–3]. Over the past two decades, many topology optimization methods have been developed, including the homogenization method [4], the SIMP (Solid Isotropic Material with Penalization) method [5,6], the level set method [7–9], the ESO (Evolutionary Structural Optimization) method [10] and other alternative methods [11–14]. Moreover, this technique has been extended to a variety of applications, such as the design of compliant mechanisms [15] and smart structures [16].

In the popular material distribution topology optimization framework, the quality of the optimal solution (the resolution and clearness of the design boundary and the analysis accuracy of the strain) can be improved by using a finer mesh. However, this greatly increases the computational cost, which may grow rapidly and become prohibitively large in practical design problems. Generally, there are two dominant factors that determine the computational cost involved in the optimization process, namely the number of finite elements and the number of design variables. Since there are usually a large percentage of regions with fully solid and void phases in the optimal solution, it is unnecessary and uneconomical to place a uniform-dense set of finite elements and design variables all over the design domain. Thus, a more efficient strategy is to start from a relatively coarse analysis mesh and density grid and then adaptively refine them only within the certain regions when necessary.

Many attempts have been made to implement adaptive topology optimization for improving the numerical efficiency and the boundary description quality of the optimal solutions by using only a necessary number of finite elements and design variables. For instance, Arantes Costa Jr and Alves [17] presented an adaptive topology optimization method with mesh refinements within and around solid regions based on the error estimation in terms of the strain field. Stainko [18] performed the refinements only along the interfaces between the fully solid and void phases. Bruggi and Verani [19] proposed a fully adaptive scheme for topology optimization based on geometric and compliance error indicators, in which both the refinement and coarsening steps were performed in the optimization to reduce the overall computational cost. Wallin et al. [20] employed an adaptive mesh strategy to resolve the interface regions in the phase-field topology optimization method. In the above adaptive approaches, the refinements are implemented on an identical mesh, which is used for discretization of both the displacement field and the material density field. Maute and Ramm [21] proposed an adaptive topology optimization

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method based on the division of analysis mesh and density field, and a smoothing function was required for mapping of the densities between these two fields. In the work of Guest and Genut [22], the design variables were adaptively activated and deactivated to reduce the number of design variables actually fed into the optimizer. Nguyen et al. [23] proposed a so-called adaptive multi-resolution topology optimization method to achieve high fidelity designs with a relatively low computational cost. Recently, Wang et al. [24] proposed an analysis mesh-independent density-point refinement concept for topology optimization. It is noted that the aforementioned adaptive methods either only refine the density field or use a same refinement criterion for both the density and the displacement fields.

As a matter of fact, from the computational efficiency point of view, it is unnecessary and uneconomic to bind the refinements of the displacement field and the density field together. Firstly, a sufficient accuracy of the displacement field is a primary concern in the structural analysis. Thus, the refinements of the analysis mesh should be implemented to improve the numerical accuracy, especially in local regions with high stress concentration or around concentrated loading points and displacement restrictions. Secondly, the material distribution topology optimization methods inherently yield intermediate densities in the local regions between the fully solid and void phases (gray transitional regions), and too much percentage of gray areas may result in undesired boundary shape error when interpreting the final optimal result. Refinements of the density field should thus be performed to improve the boundary description quality and to reduce the percentage of the gray transitional regions. Therefore, it is more reasonable to independently implement the refinements of the two separated fields according to different purposed refinement criteria. However, this issue has not yet been addressed in the literature.

In this paper, we propose a new adaptive refinement approach for topology optimization by using independent error control for the separated displacement and density fields. Two indicators, namely the energy error indicator and the gray transitional region (GTR) indicator, are used to measure the analysis accuracy and the boundary description quality, respectively. According to the different refinement criteria, the refinements are independently performed on the two separated fields, with the aim (1) to improve the analysis accuracy of local regions and to ensure the numerical stability of the optimal solutions by refining the displacement field, and (2) to improve the boundary description quality by refining the material density field. With such an adaptive strategy, the refinements of the displacement and density fields are naturally separated and no longer bond together. In this manner, the proposed adaptive method can flexibly perform the refinements on each individual field only when and where necessary.

In the proposed method, we construct the density field with point-wise density design variables, by using a strict-sense density interpolation scheme [25]. Since the work of [26], nodal design variable-based methods have attracted much attention [27–30]. Owing to their ability to represent spatial variation of material density within each element, the geometrical quality of the design boundary of the optimal structures can be improved. Recently, Kang and Wang [25] proposed an independent Point-wise Density Interpolation (iPDI) approach for topology optimization, in which an analysis mesh-independent interpolation scheme is implemented over a prescribed set of fixed density points. This interpolation scheme enables topology optimization to be conducted on fully separated density points and analysis mesh. It is therefore employed to construct the density field over the design domain in the proposed adaptive topology optimization method.

In addition to the ability of yielding high quality design boundaries and improving the computational accuracy of local regions, the proposed adaptive approach also offers other advantages. To achieve higher numerical accuracy, conventional topology optimization methods without using adaptive schemes will require globally fine analysis mesh and a large number of density design variables. However, the proposed adaptive topology optimization approach only requires relatively sparse analysis mesh, as well as a much smaller number of density design variables. Consequently, the computational cost involved in solving the equilibrium equations, computing the sensitivities and updating the design variables can be considerably reduced. Moreover, due to the newly-added density points around the boundaries between the fully solid and void phases, the gray transitional regions can be effectively narrowed and therefore higher boundary description quality can be achieved. This approach is also applicable to problems with arbitrary-shaped design domains and any type of analysis meshes.

2. Topology optimization with mesh-independent density field

2.1. Construction of material density field

In the present method, the material density field is constructed by the iPDI method [25]. Initially, the design variables are defined at a given set of regularly distributed points (density points), which are arranged independent of the analysis mesh. For a given computational point x in the design domain Ω, a circular region with the cut-off radius R_c is defined as its influence domain, as schematically illustrated in Fig. 1. Assume we have N^{dens} design variable points in the design domain, with the density values ρ_i $(i = 1, 2, \ldots, N^{\text{dens}})$, then the density at the computational point x is related to the design variables within its influence domain by [25]

$$\rho(x) = \sum_{i \in S_x} \Theta_i(x) \rho_i \tag{1}$$

where $\Theta_i(x)$ is a Shepard-family interpolation function expressed by

$$\Theta_i(x) = \frac{d(x - x_i)}{\sum_{j \in S_x} d(x - x_j)} \quad (i \in S_x) \tag{2}$$

with $d(x - x_i) = \|x - x_i\|^{-2}$ $(i = 1, 2, \ldots, N^{\text{dens}})$, and the set S_x denoting the density points located within the influence domain of point

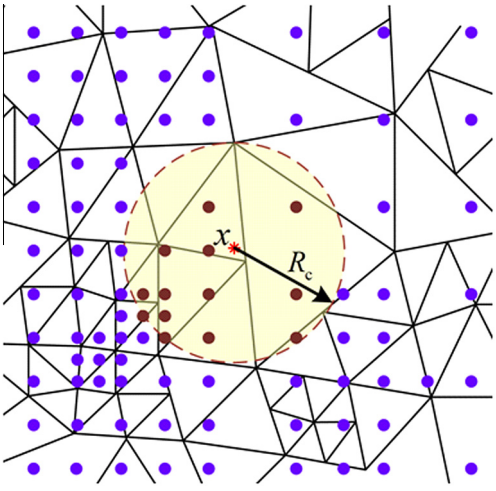

Fig. 1. Density points (design variable points) are analysis mesh-independent, and only those within the influence domain contribute to the density value of point x [25]. The brown dots represent the density points within the influence domain, while the purple ones stand for those outside. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)