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Analysis of composite plates through cell-based smoothed finite element and 4-noded mixed interpolation of tensorial components techniques

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ABSTRACT

The static bending and the free vibration analysis of composite plates are performed with Carrera's Unified Formulation (CUF). We combine the cell-based smoothed finite element method (CSFEM) and the 4-noded mixed interpolation of tensorial components approach (MITC4). The smoothing method is used for the approximation of the bending strains, whilst the mixed interpolation allows the calculation of the shear transverse stress in a different manner. With a few numerical examples, the accuracy and the efficiency of the approach is demonstrated. The insensitiveness to shear locking is also demonstrated.

1. Introduction

Increasingly complex composite structures implies complex and effective means of analysis. Different approaches can be used in the study of laminated composite structures [1–4]. In recent years, two-dimensional (2D) theories using higher-order displacement functions had proven to be a true alternative to the computationally very expensive 3D models. The theories can be equivalent-single-layer (ESL) or layerwise [5]. For the general description of 2D formulations for multilayered plates and shells, a Unified Formulation was derived by Carrera, called 'Carrera Unified Formulation' (CUF) [6–8]. This formulation is a powerful tool to implement in a single software a large number of 2D model theories, ranging from ESL models to higher layerwise descriptions. The CUF can be used in the Finite Element Method (FEM) environment [8,9] or with meshless methods [10].

Nevertheless, even with the very useful CUF, there is an important shortcoming of the FEM. For thin structures, the inclusion of both the bending and the shear stiffness in a unique rotational degree of freedom cause the locking of the finite element solution, leading to inaccurate numerical results. This shear locking phenomena can be alleviated by the use of some techniques: taking optimal rules of integration [11]; using the assumed strain method [12,13]; using field redistributed shape functions [9]; using the mixed interpolation of tensorial components (MITC) technique and by incorporating the strain smoothing technique (SFEM) [14–19]. Alternative mixed methods with similar good results can be found in Moleiro et al. [20–24].

Another approach for the elimination of the shear locking phenomena is a combination of the previous remedies. Therefore, in this work a combination of the cell-based finite element method (CSFEM) with the 4-noded quadrilateral mixed interpolation of tensorial components technique (MITC4) and selective integration rule, is considered to study the global response of laminated composites within the CUF framework. By combining different technique, we take the advantage of each technique and aim to formulate an efficient and accurate methodology, that is free from shear locking syndrome. The displacements are approximated through a sinusoidal deformation theory, and a complete study of the influence of various model parameters is performed.

The paper is organized as follows. Section 2 introduces the cell-based smoothed finite element method. In Section 3, the shear strain field according to the 4-noded mixed interpolation tensorial components technique is presented. The separation between bending and shear contributions for the stiffness matrix is discussed and a brief overview of the Carrera's Unified Formulation is presented. The shear locking phenomena is discussed in Section 4. A few numerical examples are presented in Section 5 to show the

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effectiveness of the proposed approach, followed by concluding remarks in the last section.

2. Cell-based finite element method

In the strain smoothing technique, originally proposed for meshfree methods [25], later extended to FEM by Liu et al. [26], the strain field is written as a spatial average of the compatible strain field. Based on this technique, a series of smoothed finite element method (SFEM) can be derived [17]. One such SFEM is called the cell-based smoothed finite element method (CSFEM). In the CSFEM each element is subdivided into smoothing domains, called the *subcells*. Over each subcell, the smoothing technique is performed. By judicious choice of the smoothing function and by applying the Gauss divergence theory, the surface integrals can be transformed into line integrals around the boundary of the elements in 2D. This circumvents the need to compute the derivatives of shape functions, normally required in the partition of unity framework. The strain field, $\tilde{\varepsilon}_{ij}^h$ used to compute the stiffness matrix, is computed by a weighted average of the standard strain field ε_{ij}^h. At a point $\mathbf{x}_C$ in an element Ω^h, the smoothed strain field is given by:

$$\tilde{\varepsilon}_{ij}^h = \int_{\Omega^h} \varepsilon_{ij}^h(\mathbf{x})\Phi(\mathbf{x}-\mathbf{x}_C)d\mathbf{x} \quad (1)$$

where $\Phi(\mathbf{x}-\mathbf{x}_C)$ is a smoothing function and is chosen to be:

$$\Phi(\mathbf{x}-\mathbf{x}_C) = \begin{cases} \frac{1}{A_C} & \mathbf{x}_C \in \Omega_C \\ 0 & \mathbf{x}_C \notin \Omega_C \end{cases} \quad (2)$$

being A_C is the area of the subcell. For more detailed discussion see Refs. [26,17,16,27,28].

3. MITC4 under Carrera's Unified Formulation

The transverse shear strains, interpolated according to the 4-noded mixed interpolation of tensorial components (MITC4) technique, assume the following shear strain field:

$$\{\varepsilon_s\} = \begin{Bmatrix} \{\varepsilon_{xz}\} \\ \{\varepsilon_{yz}\} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}(1+\xi)\varepsilon_{xz}^N + \frac{1}{2}(1-\xi)\varepsilon_{xz}^Q \\ \frac{1}{2}(1+\eta)\varepsilon_{yz}^P + \frac{1}{2}(1-\eta)\varepsilon_{yz}^M \end{Bmatrix} \quad (3)$$

where M, N, P and Q are sample points in the element as shown in Fig. 1. The stiffness matrix K is cleaved in two contributions, bending and shear:

$$[K] = [K_b] + [K_s]$$
$$[K_b] = \langle [B_b]^T [Q_b][B_b] \rangle; \quad [K_s] = \langle [B_s]^T [Q_s][B_s] \rangle \quad (4)$$

with the following notation:

$$\langle \cdots \rangle = \sum_{k=1}^{ns} \int_{V_k} (\ldots) dV_k \quad (5)$$

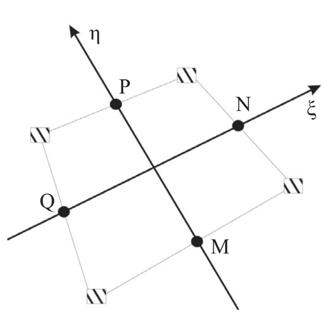

Fig. 1. Sample points (M,N,P,Q) to approximate the shear contribution in the MITC4 element.

The numerical code reflects this separation between the bending and the shear strains, within the framework of Carrera's Unified Formulation (CUF). According to CUF the displacements are expressed as a set of thickness functions depending only on the thickness coordinate z. So that the displacement field $\boldsymbol{u}(x,y,z)$ and its variation $\delta\boldsymbol{u}(x,y,z)$ are written according to the following general expansion:

$$\boldsymbol{u}(x,y,z) = F_\tau(z)\boldsymbol{u}_\tau(x,y),$$
$$\delta\boldsymbol{u}(x,y,z) = F_s(z)\delta\boldsymbol{u}_s(x,y), \quad \text{with} \quad \tau, s = 1,\ldots,N \quad (6)$$

where the summing convention with repeated indexes τ and s is assumed. F_τ and F_s are the so-called the thickness functions and they can be generic functions of the coordinate z.

In this work, the derivation of the governing equations is based on the *Principle of Virtual Displacements* (PVD) in case of a multilayered plate subjected to mechanical loads. The CUF permits us to obtain the governing equations in terms of the so-called *fundamental nuclei*, which are simple matrices representing the basic element from which the stiffness matrix of the whole structure can be computed. The PVD for a plate with N_l layers, under mechanical loads, reads:

$$\sum_{k=1}^{N_l} \delta L_{int}^k = \sum_{k=1}^{N_l} \int_{\Omega_k} \int_{A_k} \left\{ \delta\boldsymbol{\varepsilon}_{pG}^{k\ T} \boldsymbol{\sigma}_{pC}^k + \delta\boldsymbol{\varepsilon}_{nG}^{k\ T} \boldsymbol{\sigma}_{nC}^k \right\} d\Omega_k dz = \sum_{k=1}^{N_l} \delta L_e^k \quad (7)$$

where Ω_k and A_k are the integration domains in plane (x,y) and z direction, respectively, k indicates the layer and T the transpose of a vector. δL_e^k is the external work for the k^{th} layer, G implies geometrical relations and C the constitutive relations.The first step to derive the governing equations is the substitution of *constitutive equations* (C) in the variational statement PVD:

$$\boldsymbol{\sigma}_{pC}^k = \boldsymbol{C}_{pp}^k \boldsymbol{\varepsilon}_{pG}^k + \boldsymbol{C}_{pn}^k \boldsymbol{\varepsilon}_{nG}^k$$
$$\boldsymbol{\sigma}_{nC}^k = \boldsymbol{C}_{np}^k \boldsymbol{\varepsilon}_{pG}^k + \boldsymbol{C}_{nn}^k \boldsymbol{\varepsilon}_{nG}^k \quad (8)$$

with

$$\boldsymbol{C}_{pp}^k = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix} \quad \boldsymbol{C}_{pn}^k = \begin{bmatrix} 0 & 0 & C_{13} \\ 0 & 0 & C_{23} \\ 0 & 0 & C_{36} \end{bmatrix}$$
$$\boldsymbol{C}_{np}^k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{13} & C_{23} & C_{36} \end{bmatrix} \quad \boldsymbol{C}_{nn}^k = \begin{bmatrix} C_{55} & C_{45} & 0 \\ C_{45} & C_{44} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \quad (9)$$

The second step is the substitution of *geometrical relations* which relate the strains to the displacement components $\boldsymbol{u} = (u_x, u_y, u_z)$:

$$\boldsymbol{\varepsilon}_{pG}^k = \boldsymbol{D}_p^k \boldsymbol{u}^k,$$
$$\boldsymbol{\varepsilon}_{nG}^k = (\boldsymbol{D}_{n\Omega}^k + \boldsymbol{D}_{nz}^k)\boldsymbol{u}^k \quad (10)$$

wherein the differential operator arrays are defined as follows:

$$\boldsymbol{D}_p^k = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ \partial_y & \partial_x & 0 \end{bmatrix}, \quad \boldsymbol{D}_{n\Omega}^k = \begin{bmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_y \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{D}_{nz}^k = \begin{bmatrix} \partial_z & 0 & 0 \\ 0 & \partial_z & 0 \\ 0 & 0 & \partial_z \end{bmatrix}. \quad (11)$$

By introducing the Unified Formulation for the displacements, one has:

$$\boldsymbol{\varepsilon}_{pG}^k = \boldsymbol{D}_p^k(F_\tau \boldsymbol{u}_\tau^k),$$
$$\boldsymbol{\varepsilon}_{nG}^k = (\boldsymbol{D}_{n\Omega}^k + \boldsymbol{D}_{nz}^k)(F_\tau \boldsymbol{u}_\tau^k) = \boldsymbol{D}_{n\Omega}^k(F_\tau \boldsymbol{u}_\tau^k) + F_{\tau,z}\boldsymbol{u}_\tau^k \quad (12)$$

Finally, it is possible to express the displacement $\boldsymbol{u}_\tau^k$ as a function of *nodal displacements* $\boldsymbol{q}_{\tau i}^k$, by means of the *shape functions*:

$$\boldsymbol{u}_\tau^k = N_i \boldsymbol{q}_{\tau i}^k \quad (i = 1,2,\ldots,N_n) \quad (13)$$