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Optimal stiffener layout of plate/shell structures by bionic growth method

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ABSTRACT

Bionic growth method, which is based on the growth mechanism of branch systems in nature, has been used as a new approach for structural topology design optimization. Currently, its application is limited because the iterative scheme in the optimization process is heuristic. This paper suggests a new approach combined with the bionic branch model and optimality criteria. Based on the Kuhn–Tucker optimality condition, an analytical iterative formula is derived. The minimum compliance problem with multi-loading condition, the maximum fundamental frequency problem and the multi-objective optimization problem are studied. Typical design examples are demonstrated to validate the effectiveness of the suggested approach. Compared with the current growth technique, the suggested approach is more effective, practicable and applicable. The results show that the new bionic growth method can effectively and flexibly deal with optimum stiffener layout design of plate/shell structures to achieve various design objectives, thus it provides a new solution approach for structural topology design optimization.

1. Introduction

It is well known that the proper distribution of stiffeners on plate/shell structure can improve the structural mechanical performance greatly, thus a considerable amount of work has been done to develop methodologies and approaches for the design problem of optimal stiffener layout of plate/shell structures. The traditional structural topology design optimization methods are usually applied to deal with the design problem, in which the homogenization method and the density method are most frequently adopted [1–6]. Krog and Olhoff [1] considered the design problem of disk and plate structures with multiple stiffness and eigenfrequency objectives, in which the effects of using microstructures of second rank and of arbitrary ranks are studied. Ansola et al. [2] suggested an approach to optimize simultaneously the geometry of the shell mid-plane as well as the layout of surface stiffeners on the shell by introducing a variable ground structure with 2-rank layered materials. Afonso et al. [3] presented an integration procedure including topology, sizing and shape optimization for stiffened plates and shells, in which both homogenization and hybrid methods were adopted to deal with the topology optimization problem. Matteo [4] suggested a simple implementation for minimum compliance optimization based on SIMP method to deal with the generation of truss-like designs to derive preliminary strut-and-tie models not only in the established bidimensional context but also in a 3D environment. Maute and Allen [5] applied the SIMP method to the layout design of stiffeners in a wing of an aircraft by considering the fluid–structure interaction. And recently Stanford et al. [6] optimized the metallic blade-stiffened panels by a SIMP-based method to consider the bulking eigenvalue and aeroelastic flutter. In addition, some new developed topology optimization methods, such as level set method, were also used as new methods to deal with the design problem in recent years (cf. [7]). However, the mentioned traditional topology design optimization methods have such disadvantage that if the layout of the material does not form a stiffener-like pattern, the design procedure may fail. Thus post processing is usually necessary to identify the real stiffener layout and sizes, which results in difficulties in practical design, such as additional cost, and non-real optimum.

In order to overcome the disadvantages of the traditional structural topology design optimization method in stiffener layout design of plate/shell structures, some new methods were presented. For example, Bojczuk and Szteleblak [8] suggested an approach to directly determine the coordinates of control points of stiffeners by a heuristic algorithm, in which the problem of layout and shape optimization of stiffeners in plates loaded in plane and in bending Kirchhoff's plates were considered. Although the method can avoid the post processing of identifying the real stiffener layout, it is not easy to control the coordinates of control points of stiffeners when

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the number of stiffeners is big and the layout of stiffeners is complicated. On the other hand, natural structures, such as root and branch systems of plants, vascular systems of animals, can grow and branch off automatically towards such direction that can improve the global functional performance of the natural system, such as the maximum absorption of water, or uniform distribution of nutrition or blood [9,10]. It is said that these natural systems can grow adaptively to the local environment and their growth process is an optimal process to achieve multi-objective. Based on the growth mechanism of branch system in nature, a bio-inspired method was suggested by Ding and Yamazaki [11,12], which was named the bionic growth method. The method was applied to both the vibration-proof design problem [11] and the maximum stiffness design problem [12]. Since then, similar design methods were presented and applied [13–17], in which Park et al. [13] suggested an approach to improve structural dynamic characteristics via surface-grooving technique. Similar to the bionic growth method, the grooving shape is formed by merging the neighboring small embossed elements after analyzing frequency increment sensitivities of all the surrounding embossed elements. And recently, Li et al. [17] simulated the emergence of branching pattern that has a media axis and closed pattern by copying the self-optimization of leaf veins to produce the stiffener layout for plate/shell structures. They presented an update equation based on the guide-weight criterion [18] to get the so-call ideal "balanced point" in terms of weight distribution of candidate stiffeners. It is said that the bionic growth method is effective for stiffener or groove layout design problems, but the current method has disadvantages because the iterative scheme in the stiffener growth process is heuristic, although it has the explicit physical meaning [11,12]. Due to this reason, there are several coefficients in the formula, which should be selected experientially, and may affect the design result [11]. Moreover, the iterative process cannot be terminated on the condition that both the objective function and the constraint condition reach stable values, thus the optimality of the design result cannot be confirmed. These disadvantages limit the applications of the bionic growth method, especially for the optimal problem with multi-objectives and multi-constraints. In this regard, the present study suggests a new approach based on the bionic branch model and optimality criteria to improve the effectiveness and applicability of the growth method. An analytical iterative formula is derived from the Kuhn–Tucker optimality condition. On the basis of the new analytical iterative formula, the minimum compliance problem under the single and multi-loading conditions, the maximum fundamental frequency problems and the multi-objective optimization problem are studied in detail.

The remainder of this paper is organized as follows. Design method based on the bionic branch model and the analytical iterative formula derived from the optimality criteria are presented in Section 2. The minimum compliance, maximum fundamental frequency and multi-objective design problems are studied respectively in Sections 3–5, in which the specific formulas corresponding to the mentioned design problems are given and several design examples are illustrated to verify the suggested techniques. Finally, some concluding remarks are given in Section 6.

2. Bionic growth method of stiffener layout

2.1. Basic idea of bionic growth method

It is well known that living things in nature always show excellent performances to increase their survival chances by adapting themselves to the altering growth environment. The morphology of branching systems (e.g. root system of plants) is a typical example, which depends on the growth environment. Roots tend to grow downwards, away from light and towards water to fulfill their multi-functions, such as anchorage, absorption, transportation, as well as storage, support and aeration. The morphology of root systems is dependent mainly on the growth direction, which is decided by geotropism, hydrotropism, and thigmotropism. Geotropism is utilized ingeniously by roots to extend deeply and widely in the soil so as to support the ground part and to absorb water and nutrition. Hydrotropism can be explained simply as growing towards water, which is dominant over geotropism when form the morphology of the root system. In addition, because root grows by overcoming the physical resistance of the soil, thigmotropism is another important factor to affect the morphology of a root. Root can detect the contacting direction with the other roots or objects like stone and extend itself to avoid meeting them. These regulators control the growth directions of root branches, although there is some difference among different kinds of plants [19,20].

According to the growth mechanism of natural branch systems, the bionic growth method of stiffener layout design optimization for plate/shell structures was suggested by Ding and Yamazaki [11,12]. The basic idea of the method is that if the stiffeners on plates and shells extend by obeying a similar adaptive growing and branching rule as branch systems in nature, stiffened plates and shells can achieve a certain better mechanical performance.

Consider the general stiffener layout design problem of plate/shell structures, the optimization mathematic model of which is,

$$\begin{array}{ll} \text{find} & \mathbf{A} = [A_1, A_2, \ldots, A_n]^T \\ \min & \Phi(\mathbf{A}) \\ \text{s.t.} & g(\mathbf{A}) = v - r v_0 \leqslant 0 \\ & 0 < A_{\min} \leqslant A_i \leqslant A_{\max} \quad i = 1, 2, \ldots, n \end{array}, \tag{1}$$

where $\Phi(\mathbf{A})$ is the objective function, which may be selected as one of the structural performance, such as structural compliance, function of structural vibration frequency, and so on. Parameters v and v_0 are, respectively, the final volume and the initial volume of the structure, and r is the volume fraction. A_i ($i = 1,2,3,\ldots,n$) is design variable, which is the cross sectional area of the ith stiffener. And $A_{\min}$ and $A_{\max}$ are the lower and upper limits of A_i.

To solve the design problem formulated by Eq. (1), the growth mechanism of branch systems in nature is applied and a bio-inspired growth process of stiffener layout is suggested [11,12], the main steps of which are shown in Fig. 1 and explained by the followings briefly.

2.1.2. Step 1: initialization

Firstly, a ground structure based on the geometry of the plate/shell structure needed designing is modeled. The ground structure includes two parts, one is the ground shell and another is the baby stiffeners. The baby stiffeners are formed by connecting nodes of the corresponding ground shell elements, as shown in Fig. 2(a) and (b) is the cross section of the stiffened shell structure, where stiffener can be arranged concentrically and eccentrically. It is noted that the bionic growth method are valid for two kinds of arrangements [11]. However, the following studies concentrate on the layout design of concentric stiffeners. It is noted that the cross section of the stiffener is assumed to be rectangle with the same width as the thickness of the ground plate t, thus the cross sectional areas of the stiffeners, which will be set as the design variables in the optimum design process, are only changed with the heights of stiffeners h. Secondly, several initial growth points called seeds are specified on the ground structure, the baby stiffeners around the seeds are selected as the initial active stiffeners, which have the ability to grow.