Computers and Structures 135 (2014) 109–117

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

On gear tooth stiffness evaluation

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ARTICLE INFO

Article history:
Received 23 April 2013
Accepted 15 January 2014
Available online 8 February 2014

Keywords:
Gears
Stiffness
Spur
External/internal
FE

ABSTRACT

The estimation of gear stiffness is important for determining the load distribution between the gear teeth when two sets of teeth are in contact. Two factors have a major influence on the stiffness; firstly the boundary condition through the gear rim size included in the stiffness calculation and secondly the size of the contact. In the FE calculation the true gear tooth root profile is applied. The meshing stiffnesses of gears are highly non-linear, it is however found that the stiffness of an individual tooth can be expressed in a linear form assuming that the contact width is constant.

1. Introduction

To evaluate the forces in a multibody formulation of a planetary gearbox where multiple planet gears are in contact with the ring and sun gear an algebraic constraint formulation cannot be used if the forces on the individual teeth of the involved gears are of interest. We are forced to include in some way the gear contact and here include the flexibility, as discussed in i.e., [1,2]. Including the flexibility through a full integration of a finite element (FE) formulation within the multibody simulation is not feasible due to the simulation time needed. Instead hybrid methods of lumping in some way the stiffness are needed. The present paper focuses on the estimation of gear teeth stiffnesses, these stiffnesses can then later be applied in a multibody modeling.

Gears are highly standardized and the most common gear design applied is controlled by the cutting tool or basic rack defined in [3], and seen in Fig. 1. The gear load capacity can be evaluated using [4]. The basic contacting gear shape is the involute shape, due to the excellent properties of this shape. These properties includes that the contact forces act along a straight line and that a center distance error do not influence this. A change in center distance due to, e.g., loading will neither influence the gear ratio. The design variable that controls the involute shape is the pressure angle α, see Fig. 1. The commonly used value for the pressure angle is $\alpha = \pi/9$. The involute shape controls the gear part that is in contact with the other gear in the mesh. The gear root or bottom land that connects two neighboring teeth is controlled by the cutting tool tip design, there is no contact between the teeth at the root. Different ways of modifying the root design and improving the stress concentrations can be found in [5,6].

The contact between two involute gears follows the straight contact line (the dotted line in Fig. 2) at all time during the contact. Seen from the individual tooth the contact load moves along the involute shape. Due to the design of a tooth, see Fig. 2, this results in an overall non-linear tooth stiffness along the contact line.

The tooth stiffness is needed for multiple reasons. For gears in mesh we have different number of teeth in contact during the motion. For spur gears produced from a standard rack cutter with $\alpha = \pi/9$ and a height of $2M$, i.e., two times the module (see [3] and Fig. 1) we have that the maximum contact ratio is $(\epsilon_\alpha)_{max} \approx 1.98$. The contact ratio expresses the average number of teeth in contact during the motion. With more than two sets of teeth in contact we need the tooth stiffness for finding the load on the individual tooth. This is also the case for planetary gears where multiple planet gears are in contact with the sun gear and the ring gear. The nature of the gear mesh contact is discontinuous, for standard spur gears there are at some instance in time either one or two gear teeth sets in contact and this transition result in discontinuous mesh stiffness.

From a literature study it is obvious that many different approaches have been applied. In [7] FE analysis was used for the tooth stiffness estimation. Here a model with only one tooth attached to a rim/ring with a given thickness of 2.5 times the tooth height was applied. The stiffness is calculated for two gears in contact, for one gear a torque is applied to the rim inner boundary and at the other gear the rim inner boundary is fixed. The tooth root is assumed circular. A principally different way of estimating the

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http://dx.doi.org/10.1016/j.compstruc.2014.01.023
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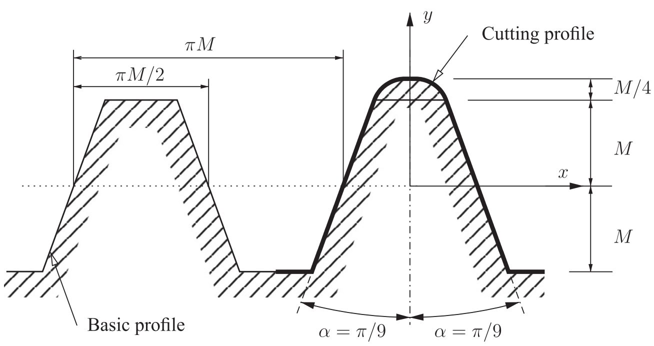

Fig. 1. The cutting profile geometric definition and the basic profile based on the ISO profile.

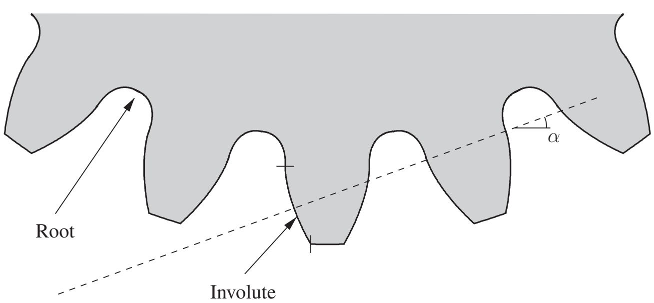

Fig. 2. Part of a gear with 20 teeth and module $M = 10$ mm. The line of contact is indicated by the dotted line, the angle is α assuming that no profile shift have been applied to both gears in mesh.

stiffness is to use the elastic energy. Using the elastic energy and the applied force for specifying the stiffness is clear and concise, the need for finding a deflection corresponding to the force is avoided. This way of establishing the stiffness is used in relation to bolt plate stiffness in [8–10] and is also the selected method in the present paper. In [11] the stiffness is found using an analytical estimation of the complementary elastic energy, several assumptions are applied in this work. Further stiffness evaluation of spur and helical gears can be found in, e.g., [12–17].

In the present paper two factors are found to have a major influence on the gear tooth stiffness, these are

- The gear rim size included in the stiffness calculation.
- The contact zone size.

To the authors knowledge no thorough examination have been published in relation to these two factors, even for the simple case of spur gears. The bending strength calculations of most standards rely on the cantilever beam assumption (Timoshenko theory) and the tooth is clamped at the root. This is also the general choice for estimating the stiffness either by analytical method or numerical FE calculations. The present paper will focus on the definition of boundary conditions, and the influence from the contact zone width.

The paper presents an example where the found stiffnesses are used in the simulation of a planetary gearbox of a 500 MW wind turbine. Modeling specifically related to planetary gearbox can be found in, e.g., [18] and in relation to wind turbine drive train see e.g., [19,20].

The paper is organized as follows. Section 2 presents the geometric description and the tooth boundary parameterization. In Section 3 the individual tooth stiffness is given. The section includes a discussion on the important selection of boundary condition and contact width. The combined stiffness of two gear teeth in contact and the overall meshing stiffness are found. Section 4 presents an example where stiffnesses found using the proposed approach is used in a planetary gearbox of a wind turbine. Finally a generic gear tooth geometry determination in parametric form is presented in the Appendix A.

2. Parametric geometry description

The ISO profile gear geometry is controlled by the cutting tools outer profile design as presented in Fig. 1. The standard profile is defined by connected curve segments (arc of circles and straight lines). A procedure for finding the curve segments can be found in [5,6] and in the appendix of the present paper.

The gear tooth contact geometry is the envelope defined by rotating and simultaneously translating the straight side of the cutting tool. For this segment we know that the envelope corresponds to the involute shape, which for the curve given in Fig. 3 is given as a function of s by

$$\begin{Bmatrix} x(s/r_b) \\ y(s/r_b) \end{Bmatrix} = \begin{bmatrix} \cos(s/r_b) & -\sin(s/r_b) \\ \sin(s/r_b) & \cos(s/r_b) \end{bmatrix} \begin{Bmatrix} r_b \\ -s \end{Bmatrix} \tag{1}$$

where r_b is the base circle radius, the parameter s is the base circle arc length which is directly related to the involute arc length. The base circle radius is given by

$$r_b = M\frac{z}{2}\cos(\alpha) \tag{2}$$

where z is the number of gear teeth.

We may use (1) as a specific alternative to the general parameterization presented in the appendix.

The present papers primary objective is finding the tooth stiffness as a function of the loading point on the gear. In [11] the load position is defined by a profile parameter defined as

$$\xi = \frac{z}{2\pi}\sqrt{\frac{r_c^2}{r_b^2} - 1} \tag{3}$$

where r_c is the distance from the gear center to the contact point (see Fig. 3). We directly see that

$$s = \sqrt{r_c^2 - r_b^2} \tag{4}$$

$$\xi = s\frac{z}{2\pi r_b} = \frac{s}{\pi M\cos(\alpha)} = \frac{s}{s_d} \tag{5}$$

here we have defined the contact pitch $s_d = \pi M\cos(\alpha)$, this distance is only a function of the pressure angle α and the module.

The stiffness will in the present paper be given as a function of the base circle arc length s, from (4) and (5) follows that it can directly be scaled to be given as a function of r_c or ξ. The base circle

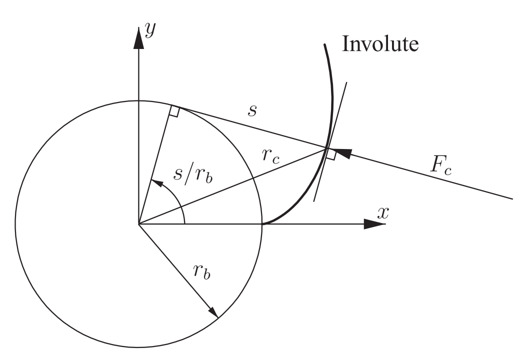

Fig. 3. Base circle and involute geometry, the angle is defined by the base circle arc length s. An arbitrary contact load F_c is shown to be perpendicular to the involute shape.