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Multi-constrained 3D topology optimization via augmented topological level-set

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ABSTRACT

The objective of this paper is to introduce and demonstrate a robust methodology for solving multi-constrained 3D topology optimization problems. The proposed methodology is a combination of the topological level-set formulation, augmented Lagrangian algorithm, and assembly-free deflated finite element analysis (FEA).

The salient features of the proposed method include: (1) it exploits the topological sensitivity fields that can be derived for a variety of constraints, (2) it rests on well-established augmented Lagrangian formulation to solve constrained problems, and (3) it overcomes the computational challenges by employing assembly-free deflated FEA. The proposed method is illustrated through several 3D numerical experiments.

1. Introduction

Over the last two decades, topology optimization (TO) [1] has accelerated from an academic exercise into an exciting discipline with, potentially, numerous industrial applications. The focus of this paper is specifically on *constrained* TO where several performance and manufacturing constraints must be considered during optimization.

In structural mechanics, a constrained TO problem may be posed as (see Fig. 1):

$$\begin{aligned} &\underset{\Omega\subset D}{Min}\ \varphi(u,\Omega) \\ &g_i(u,\Omega)\leqslant 0;\quad i=1,2,\ldots,m \\ &\text{subject to} \\ &Ku=f \end{aligned} \tag{1.1}$$

where:

φ : Objective to be minimized
Ω : Topology to be computed
D : Domain within which the topology must lie
u : Finite element displacement field
K : Finite element stiffness matrix
f : External force vector
g_i : Constraints
m : Number of constraints

Various methods have been proposed to solve specific instances of Eq. (1.1); these are reviewed in Section 2. For example, a special case of Eq. (1.1) is the *compliance-constrained* volume minimization problem:

$$\begin{aligned} &\underset{\Omega\subset D}{Min}\ |\Omega| \\ &J\leqslant J_{all} \\ &\text{subject to} \\ &Ku=f \end{aligned} \tag{1.3}$$

where

$$\begin{aligned} &J:\ \text{Compliance} \\ &J_{all}:\ \text{Compliance allowable} \end{aligned} \tag{1.4}$$

Fig. 2 illustrates the solution to a specific instance of Eq. (1.3), where the allowable compliance is 60% larger than the initial compliance.

In practice, additional constraints including stress, buckling, Eigen-value, and manufacturing constraints must be taken into account. The objective of this paper is to develop a unified method that can solve such multi-constrained TO problems. The proposed method and its implementation are discussed in Section 3. In Section 4, numerical experiments are presented, followed by conclusions in Section 5.

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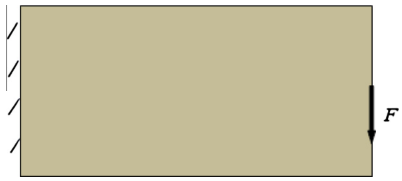

Fig. 1. A single-load structural problem.

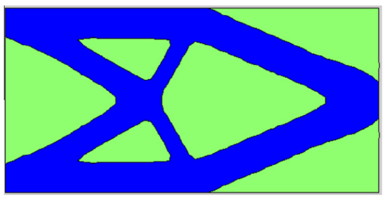

Fig. 2. Optimal topology for a specific instance of Eq. (1.3) over the structure in Fig. 1.

2. Literature review

2.1. Constrained topology optimization

To solve a constrained TO problem, a TO formulation and a constrained optimization algorithm must be chosen.

Various *TO formulations* including homogenization [2], Solid Isotropic Material with Penalization (SIMP) [3] and level-set [4,5], have been proposed. Constrained optimization algorithms, on the other hand, include method of moving asymptotes (MMA) [6], optimality criteria (OC) [7], simplex method [8], interior point method [9], Lagrangian multiplier method [9], augmented Lagrangian method [9] and so on.

We review below various combinations of TO formulations and optimization algorithms that have been proposed. Table 1 provides a chronological summary of relevant literature. The table and the review that follows are representative but not exhaustive; for example, constrained ground structures methods [10–12] are not reviewed here.

2.1.1. SIMP based methods

SIMP is perhaps the most popular TO formulation due to its simplicity, generality and success in several applications [13]. Based on the finite element method (FEM), SIMP assigns each element with a pseudo-density, and the pseudo-densities are then optimized to meet the desired objective [14].

Initially, SIMP was employed to solve compliance minimization problems [14]; it then evolved to include constraints. For example, one of the earliest SIMP-based stress-constrained TO implementation was reported in [15] where authors coalesced local stress constraints into a global stress constraint, and addressed instability issues via a weighted combination of compliance and global stress constraints. Further research on compliance and stress-constrained SIMP-based TO are discussed in [13,16–19].

In [20], the authors proposed a SIMP-based trust-region method combined with augmented Lagrangian to solve a TO problem of continuum structures subject to failure constraints. In [21], a Heaviside design parameterization was used in SIMP to consider manufacturing constraints. The authors in [22] implemented SIMP with MMA to solve a TO problem with compliance and manufacturing constraints. In [23], using SIMP, a manufacturing constraint and a unilateral contact constraint were absorbed into compliance minimization formulation through augmented Lagrangian method. In [24], the authors used a modified SIMP formulation coupled with quadratic programming technique to minimize structural weight subject to multiple displacement constraints. The authors in [25] used MMA to solve a topology optimization problem with a probability-based high-cycle fatigue constraint. In [26], an algorithm was proposed to address multi-scale topology optimization problems subject to multiple material design constraints. In [27], a multi-point approximation algorithm was used as optimizer in a continuum structure topology optimization problem subject to dynamic constraints.

2.1.2. ESO/BESO based methods

ESO [28] is an alternate TO formulation where finite elements are gradually removed during each iteration. BESO [29] addresses some of the limitations of ESO by permitting the insertion of elements.

In [30], a principal-stress based ESO method was proposed to find the optimal design of cable-supported bridges subject to displacement and frequency constraints. During each optimization iteration, based on a threshold, elements were removed from the design domain. A similar method was used in [31] to solve contact design problems, where the authors proposed the interfacial gap between components be treated as optimization variables, while the contact stress be treated as an objective function. In [32], the Lagrangian multiplier method was used with BESO to combine the objective function of structural stiffness with a local displacement constraint. In [33], a modified BESO method was combined with optimality criteria to solve a topology optimization problem with natural frequency constraints. The authors argued this method can successfully avoid artificial local modes.

2.1.3. Level-set based methods

Level-set formulation is gaining popularity in TO since it permits an unambiguous description of the boundary, and therefore permits imposition of constraints on the boundary. The level-set formulation relies on an evolving level-set which is typically controlled via Hamilton–Jacobi equations [34]. Readers are referred to [34] for a recent review of the success of level-set based methods in structural TO.

In [35], X-FEM based level-set and OC method were combined to find optimal designs for continuum structures with geometric constraints. In [36], a topological level-set method was coupled with an adapted weight method for solving stress-constrained compliance minimization problem. In [37], the authors combined classic shape derivative and level set method for front propagation; the Lagrangian multiplier technique was used for perimeter-control. Since there was no implemented mechanism for creation of holes, the final design was dependent on initial material layout. In [38], the augmented Lagrangian technique was combined with the topological sensitivity based level-set method to handle displacement, stress and compliance constraints.

In [39], level-set/X-FEM combined with a shape equilibrium constraint strategy was proposed. Specifically, a TO problem with stress constraint was formulated through Lagrangian multiplier method which was then iteratively solved. In [40], a level-set based method was derived to handle casting constraints; augmented Lagrangian method was applied for posing the constraints and calculating the shape derivative of objective function. In [41], a level-set based method was applied to the representative wing box of NASA Common Research Model to find the optimal 3-D aircraft wing structures. Compliance was minimized while balancing the aerodynamic lift and total weight. The level-set was shown to be robust and efficient by finding optimum solutions for multiple aerodynamic and body force load cases.