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A numerical method to generate optimal load paths in plain and reinforced concrete structures

Matteo Bruggi

Department of Civil and Environmental Engineering, Politecnico di Milano, I20133 Milano, Italy

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ABSTRACT

A numerical method based on topology optimization is proposed to generate optimal strut-only models for structures made of plain concrete and optimal strut-and-tie models for concrete structures where fixed regions of reinforcement are prescribed. Assuming concrete as a hyper-elastic material carrying only compression, both the inherently nonlinear equilibrium equation and the energy-based topology optimization problem are solved within the same minimization procedure. Numerical simulations investigate load paths within the two-dimensional domain in case of conventional rebar cages. A stress diffusion problem is considered as well.

1. Introduction

Strut-and-tie models (STMs) are of primary importance when designing reinforced concrete members. This detailing approach investigates truss-like structures to define statically admissible load paths that connect the load application points to the ground constraints, see e.g. [1]. It was firstly introduced to cope with the transversal reinforcement for shear design of beams [2,3] and then extended to the so-called D-regions of reinforced concrete structures, where the Bernoulli hypothesis does not hold due to any discontinuity in the geometry of the specimen or in the load pattern [4]. A complex strain/stress state arises in these regions, meaning that the conventional assumption about cross sections remaining plane after bending falls. This implies that standard design methods based on the beam theory cannot be applied. According to the plasticity theorem, any statically admissible truss-like structure that does not violate the yield criteria has a load-carrying capacity that is a lower-bound of the ultimate strength of the specimen. This supports the adoption of strut-and-tie modeling to define statically admissible load paths for a proper detailing of the D-regions.

Methods of structural optimization have been used to generate suitable strut-and-tie models among the number of truss-like structures that can be defined within a certain domain. The ground structure approach was firstly adopted to select minimum energy or minimum weight strut-and-tie models addressing grids of discrete trusses, see e.g. [5–8]. Afterwards, topology optimization was introduced to generate optimal truss-like layouts adopting the same objective functions but resorting to the finite element modeling for plane problems. Evolutionary structural optimization was used in the pioneering approaches proposed in [9–13], whereas the distribution of solid isotropic material according to the SIMP model for stiffness penalization [14] was adopted e.g. in [15–20]. The latter approach has also been used for the conceptual design of optimal layouts of steel reinforcement dropping the conventional strut-and-tie approach and moving towards more general problems of topology optimization for reinforced concrete structures, involving non-symmetric strength constraints, see e.g. [21–23], damage mechanics [24,25], elastoplasticity, see [26]. Remaining in the field of strut-and-tie modeling, an alternative approach combining truss design and topology optimization has been recently presented in [27,28] to look for load paths that are not only statically admissible but also consistent with the stress flow in the concrete element.

To ensure appropriate safety standards, designers are generally required by technical codes to neglect the limited tensile strength of concrete when detailing reinforced concrete beam sections at the ultimate limit state. Indeed, strut-and-tie modeling stands on the assumption that concrete carries compressive stresses only, whereas steel is used as a reinforcement to carry any tensile stress. According to technical rules, best STMs can be selected through energy-based criteria [29]. These prescriptions explain the strong interest of the community of designers towards the adoption of simple numerical methods exploiting topology optimization to distribute linear elastic isotropic material and achieve truss-like layouts as preferred strut-and-tie load paths, see [30]. Assessment

E-mail address: matteo.bruggi@polimi.it

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of the achieved results and detailing of each member making the optimal STM can be performed subsequently through ad hoc tools, see e.g. [31].

Unfortunately, the linear elastic approach fails when addressing the achievement of energy-based load paths in structures where the layout of the reinforcement is already prescribed (due to the adoption of a fixed rebar cage) or when copying with the optimal design of plain concrete members (with no rebar allowed except for the secondary reinforcement intended for ductility and durability requirements only). In these cases, conventional methods of topology optimization should be modified to avoid the arising of any tensile-stressed member within the concrete domain. This can be conveniently pursued by implementing methods of energy-based topology optimization for compression-only materials.

Alternative formulations have been proposed in the last decades to address tension-only or compression-only materials in topology optimization. Most of them resort to non-linear modeling, see e.g. [32], or to re-modeling theories and material replacement strategies that distribute the unilateral material depending on the directions of the stress flows computed in the design domain, see e.g. [33,17,34]. A simplified stress-based approach has been presented in [35] that implements a smooth approximation of the unilateral condition through the formulation of a suitable version of the Drucker–Prager strength criterion. Due to the inherent isotropic modeling, this approach cannot be straightforwardly applied to structures governed by any complex biaxial stress state. In this case, a full no-tension/no-compression modeling of the constitutive behavior of the material should be implemented.

Hyper-elasticity of unilateral materials allows solving the inherently nonlinear equilibrium through the minimization of the strain energy, see in particular [36]. The energy-based approach presented in [37] is herein exploited to address the generation of optimal STMs through a topology optimization problem that copes with concrete as a compression-only material. An equivalent orthotropic medium is defined, exhibiting negligible stiffness for any direction along which a tensile principal stress must be prevented. Two sets of density unknowns are needed to control the stiffness of the equivalent composite along its symmetry axes, which should be oriented along the principal stress directions of the no-tension body. The topology optimization problem adopts the weight as objective function, whereas a global constraint is prescribed on the overall compliance of the specimen (made of plain concrete or concrete reinforced by a fixed rebar cage). Its solution is performed adopting an established method of sequential convex programming, the Method of Moving Asymptotes MMA [38]. The compression-only behavior is efficiently enforced through an ad hoc penalization of the energy terms related to any tensile stress arising during the optimization.

The layout of the paper is as follows. Section 2 recalls the rationale used to model a compression-only solid as an equivalent orthotropic medium, whereas Section 3 introduces the energy-based topology optimization problem that is used to derive optimal STMs addressing concrete as a compression-only material. Section 3.1 provides details on the implementation of the solving algorithm. Features of the achieved optimal STMs are discussed in Section 4, presenting numerical examples and providing comparison with optimal layouts found through the conventional approach with linear elastic modeling and equal behavior in tension and compression of the material. Section 4.1 is devoted to optimal strut-only models for structures made of plain concrete. Section 4.2 deals with optimal strut-and-tie models for concrete structures where fixed regions of reinforcement are prescribed. Section 5 resumes the main findings of the work, outlining future extensions of this research.

2. A material interpolation for concrete

Young modulus E and non-negative Poisson's ratio ν are herein assumed for a material that can sustain only compressive stresses, dealing with plane stress conditions. The principal axis z_{III} is orthogonal to the reference plane whose coordinates are labeled z_1 and z_2. Let σ_I, σ_{II} and $\sigma_{III} = 0$ be the principal stresses for the tensor $\sigma_{ij}(\chi)$, as computed at any point $\chi \in \Omega$, with $\sigma_I \leqslant \sigma_{II}$.

The domain Ω is divided into three sub-regions such that $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3$ and:

$$\begin{aligned} \Omega_1 &= \chi \in \Omega : \sigma_I < 0, \quad \sigma_{II} < 0, \\ \Omega_2 &= \chi \in \Omega : \sigma_I < 0, \quad \sigma_{II} = 0, \\ \Omega_3 &= \chi \in \Omega : \sigma_I = 0. \end{aligned} \tag{1}$$

In sub-region Ω_1 the material is acted upon by biaxial compression and behaves as any conventional isotropic medium. In Ω_2 the material is acted upon by uniaxial compression and behaves as an orthotropic medium. A fully elastic behavior is recovered along the compressive isostatic line, whereas some inelastic strain $\varepsilon^c \geqslant 0$ arises in the orthogonal direction. In sub-region Ω_3 neither stress nor elastic strain is found and the solid behaves as a "void phase", meaning that any positive semi-definite inelastic strain is allowed.

An equivalent orthotropic material model has been formulated in [37] to describe the outlined behavior through the same closed-form expression at any point $\chi \in \Omega$. This may be done introducing two fields of density unknowns that allow specializing the behavior of the solid in each of the sub-regions Ω_1, Ω_2 and Ω_3 of Eq. (1), penalizing the stiffness of the orthotropic material along its symmetry axes $\tilde{z}_1$ and $\tilde{z}_2$. Principal stress directions z_I and z_{II} and the symmetry axes $\tilde{z}_1$ and $\tilde{z}_2$ share the same orientation, as provided by the angle θ, see Fig. 1. Referring to the elastic constants of the equivalent orthotropic medium, $\tilde{E}_1$, $\tilde{E}_2$ are the Young moduli of the material (along $\tilde{z}_1$ and $\tilde{z}_2$, respectively), $\tilde{G}_{12}$ is the shear modulus and $\tilde{\nu}_{12}$, $\tilde{\nu}_{21}$ are the Poisson's ratios. It is recalled that the equality $\tilde{\nu}_{ij}\tilde{E}_j = \tilde{\nu}_{ji}\tilde{E}_i$ holds.

The design problem will be solved through a displacement-based finite element method, adopting four node bilinear elements. In view of the adoption of such a discretization, let denote as x_{1e} and x_{2e} the density unknowns that govern the behavior of the equivalent composite along its symmetry axes (i.e. the isostatic stress lines of the compression-only material) related to each element of the mesh. Following a generalization of the SIMP (Solid Isotropic Material with Penalization) [14,39] used in topology optimization for isotropic materials, the elastic constants for the equivalent orthotropic material can be written as:

$$\tilde{E}_i = x_{ie}^p E, \quad \tilde{G}_{ij} = x_{ie}^p x_{je}^p \frac{E}{2(1+\nu)}, \quad \tilde{\nu}_{ij} = x_{je}^p \nu, \quad \text{for } i,\ j = 1,2. \tag{2}$$

Reference is made to [40,41] for the original development of the above formula within the framework of derivable optimality criteria methods for structural optimization. As usually done in conventional formulations for the topology optimization of isotropic materials the penalization parameter is assumed such that $p = 3$, see in particular [30]. The interpolation in Eq. (2) is especially conceived to provide vanishing stiffness along any direction along which the relevant density unknown, x_{1e} or x_{2e}, takes its minimum value. In the general reference with axes z_1 and z_2, denoting by $\underline{\sigma}_e = [\sigma_{11}\ \sigma_{22}\ \sigma_{12}]$ the vector of the stress components for the e-th element and by $\underline{\varepsilon}_e = [\varepsilon_{11}\ \varepsilon_{22}\ 2\varepsilon_{12}]$ the corresponding strain components, the constitutive law for the equivalent orthotropic material reads:

$$\underline{\sigma}_e = \mathbf{T}(\theta_e)^{-1}\mathbf{C}(x_{1e}, x_{2e})\mathbf{T}(\theta_e)^{-t}\underline{\varepsilon}_e, \tag{3}$$