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Pure jump models for pricing and hedging VIX derivatives

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ABSTRACT

Recent non-parametric statistical analysis of high-frequency VIX data (Todorov and Tauchen, 2011) reveals that VIX dynamics is a pure jump semimartingale with infinite jump activity and infinite variation. To our best knowledge, existing models in the literature for pricing and hedging VIX derivatives do not have these features. This paper fills this gap by developing a novel class of parsimonious pure jump models with such features for VIX based on the additive time change technique proposed in Li et al. (2016a, 2016b). We time change the 3/2 diffusion by a class of additive subordinators with infinite activity, yielding pure jump Markov semimartingales with infinite activity and infinite variation. These processes have time and state dependent jumps that are mean reverting and are able to capture stylized features of VIX. Our models take the initial term structure of VIX futures as input and are analytically tractable for pricing VIX futures and European options via eigenfunction expansions. Through calibration exercises, we show that our model is able to achieve excellent fit for the VIX implied volatility surface which typically exhibits very steep skews. Comparison to two other models in terms of calibration reveals that our model performs better both in-sample and out-of-sample. We explain the ability of our model to fit the volatility surface by evaluating the matching of moments implied from market VIX option prices. To hedge VIX options, we develop a dynamic strategy which minimizes instantaneous jump risk at each rebalancing time while controlling transaction cost. Its effectiveness is demonstrated through a simulation study on hedging Bermudan style VIX options.

1. Introduction

Originally introduced in 1993 and redefined in 2003, the CBOE volatility index, VIX, has become a standard measure of volatility risk in financial markets. VIX is designed to reflect the market's expectation of volatility in the next 30 days and is commonly regarded as the fear gauge. The trading of VIX futures started in 2004, followed by the trading of European style VIX options in 2006. In recent years, the VIX futures and options market has been growing rapidly (see Fig. 1), and nowadays these derivatives are among the most actively traded contracts on CBOE. The popularity of VIX derivatives can be explained by the wide recognition that they are important tools for managing volatility risk, which is highlighted in the recent financial crisis. This paper considers developing stochastic models for pricing and hedging VIX derivatives.

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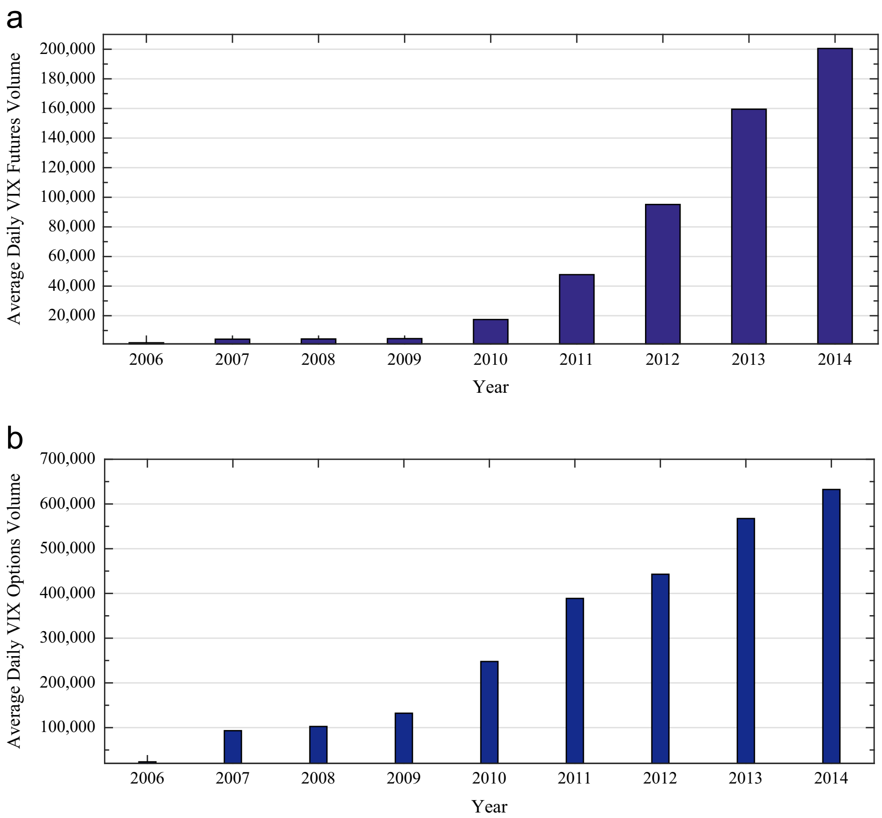

Fig. 1. Average daily trading volume of VIX futures and options by year (data source: CBOE).

There are currently two modelling approaches for pricing and hedging VIX derivatives. The first one starts with a model for SPX (e.g., Zhang and Zhu, 2006; Lin, 2007; Lin and Chang, 2009, 2010; Zhu and Lian, 2012; Baldeaux and Badran, 2014) or a joint model for the forward variance swap term structure and SPX (Gatheral, 2008; Cont and Kokholm, 2013), and then derives the dynamics for VIX. This approach allows consistent modelling between SPX and VIX, and is particularly suitable if one is interested in pricing derivatives whose payoffs depend on both SPX and VIX. However, such models may not be analytically tractable for pricing VIX options and/or may not be able to calibrate the entire VIX volatility surface and/or may not be very parsimonious. The second approach directly specifies a stochastic dynamics for VIX, which is the route we take in this paper. Our main interest here is in developing a parsimonious model for pricing and hedging various types of VIX options using VIX futures and vanilla options, with the requirements that the model captures key features of VIX, is consistent with the initial term structure of VIX futures, is analytically tractable for pricing European style VIX options to facilitate fast calibration, and is able to calibrate the VIX volatility surface. Given the depth and liquidity of the current VIX futures and options market, we view direct modelling of VIX as an appropriate approach to construct a parsimonious model that meets all these requirements (see Drimus and Farkas, 2013 which also follows the second route).

It is well documented in the literature that mean-reversion and jumps are two salient features of VIX. Existing stochastic models for VIX include mean-reverting diffusion and jump-diffusion specifications. In the family of diffusion models, Detemple and Osakwe (2000) assume VIX to follow an exponential Ornstein–Uhlenbeck diffusion while (Grünbichler and Longstaff, 1996) uses the CIR process. Drimus and Farkas (2013) extend Dupire's local volatility approach to mean-reverting diffusions with linear mean-reverting drift for VIX. Recently, Goard and Mazur (2013) test a class of mean-reverting diffusions for fitting VIX data, which includes the exponential Ornstein-Uhlenbeck process, the CIR process and the 3/2 diffusion (which is the solution to the SDE $dX_t = \kappa X_t(\theta - X_t) + \sigma X_t^{3/2} dB_t$ with $\kappa, \theta, \sigma > 0$). This paper finds that the 3/2 diffusion achieves the best fit for VIX data. Papers that employ a jump-diffusion specification for VIX include for example, Dotsis et al. (2007), Psychoyios et al. (2010), Mencía and Sentana (2013), where an independent compound Poisson jump component is added to the diffusion part.

Through non-parametric statistical analysis of high-frequency VIX data, a recent paper (Todorov and Tauchen, 2011) reveals that VIX dynamics is a pure jump semimartingale with infinite jump activity and infinite variation. Here "pure jump" means that the process is void of the continuous local martingale component (i.e., the process is not driven by any Brownian motion), and "infinite jump activity" says that there are infinite number of jumps in any finite time interval (more precisely, in any finite time interval, the number of large jumps is finite but the number of arbitrarily small jumps is infinite; see Todorov and Tauchen, 2011). In a pure jump VIX model with infinite activity, the infinite activity jump part with arbitrarily