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Bayesian estimation of agent-based models

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ABSTRACT

We consider Bayesian inference techniques for agent-based (AB) models, as an alternative to simulated minimum distance (SMD). Three computationally heavy steps are involved: (i) simulating the model, (ii) estimating the likelihood and (iii) sampling from the posterior distribution of the parameters. Computational complexity of AB models implies that efficient techniques have to be used with respect to points (ii) and (iii), possibly involving approximations. We first discuss non-parametric (kernel density) estimation of the likelihood, coupled with Markov chain Monte Carlo sampling schemes. We then turn to parametric approximations of the likelihood, which can be derived by observing the distribution of the simulation outcomes around the statistical equilibria, or by assuming a specific form for the distribution of external deviations in the data. Finally, we introduce Approximate Bayesian Computation techniques for likelihood-free estimation. These allow embedding SMD methods in a Bayesian framework, and are particularly suited when robust estimation is needed. These techniques are first tested in a simple price discovery model with one parameter, and then employed to estimate the behavioural macroeconomic model of De Grauwe (2012), with nine unknown parameters.

1. Introduction

Agent-based (AB) models are structural dynamical models characterized by three features: (i) there are a multitude of objects that interact with each other and with the environment, (ii) these objects are autonomous, that is there is no central, or 'top-down' control over their behaviour and more generally on the dynamics of the system (e.g. a Walrasian auctioneer), and (iii) aggregation is performed numerically (Richiardi, 2012). AB models are increasingly used in disciplines as diverse as geography, anthropology, sociology, biology, political science, epidemiology (Macal, 2016). In macroeconomics, they have been proposed as an alternative to dynamic stochastic general equilibrium (DSGE) models, where the tenet of rational expectations is replaced by an explicit modelling of learning and selection, thus allowing for more heterogeneity in agents' characteristics and behaviour and a more detailed account of the (physical and institutional) environment, leading to more complex interaction patterns (Richiardi, 2016).[1]

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[1] For a plea for the AB approach, and a critique of the DSGE approach to macroeconomics, see Fagiolo and Roventini (2016).

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There is a small but growing literature on estimation of AB models. This is surveyed in Grazzini and Richiardi (2015), who also show how to apply simulated minimum distance (SMD) techniques to estimate the parameters, following a frequentist approach. The method of simulated moments (MSM) and indirect inference (II), among other techniques, fall in this general class. Typically, certain summary statistics have to be selected in advance in order to implement the minimum-distance estimator, requiring additional sensitivity analysis to understand their properties. Good properties of the summary statistics ensure good properties of the estimator, e.g. consistency. Recently, Kukacka and Barunik (2016) have proposed non-parametric simulated maximum likelihood, and apply it to the estimation of the model described in Brock and Hommes (1998). Other contributions have introduced sophisticated validation methods aimed at measuring the distance between real and simulated data, where the simulated data can either come from the original AB model (Barde, 2016; Fabretti, 2013; Guerini and Moneta, 2016; Lamperti, 2015; Marks, 2013; Recchioni et al., 2015), or from a surrogate model (Salle and Yildizoglu, 2014; Sani et al., 2016). Minimisation of such distance metrics within a SMD approach leads to alternative estimators with respect to those commonly employed in the literature, such as MSM or II, although the properties of these estimators, including consistency, and their associated uncertainty have yet to be fully assessed.[2]

While the literature on calibration and SMD estimation of AB models is moving fast, the Bayesian approach common for instance in the DSGE (dynamic stochastic general equilibrium) literature has so far received less attention. In this note, we show how Bayesian methods can be used to perform statistical inference in AB models. The advantages of Bayesian methods with respect to the frequentist and the calibration approaches are twofold: (i) they do not require to pre-select moments (MSM) or an auxiliary model (II), or other metrics (calibration) to evaluate the distance between the real and the simulated time series, and (ii) they allow to incorporate prior information, leading to a proper statistical treatment of the uncertainty of our knowledge, and how it is updated given the available observations. Moreover, with respect to methods that require the use of summary statistics (as MSM), the Bayesian approach fully exploits the informational content of the data, hence achieving, at least asymptotically, greater efficiency.[3]

On the other hand, the Bayesian approach can mask identification problems (Canova, 2008; Canova and Sala, 2009) by artificially adding curvature to the posterior with appropriately selected (and often poorly justified) priors, in presence of a flat likelihood. This is a malpractice that afflicts DSGE models (Fagiolo and Roventini, 2012; 2016), but that should not let us jump to the conclusion to "throw the baby out with the bathwater". A more serious potential disadvantage of Bayesian methods, with respect to calibration techniques, is the computational burden of estimating the likelihood function by simulation. To save on these computational costs, in addition to using efficient sampling schemes in models with large parameters' space, several approximations can be introduced, whose appropriateness should be evaluated on a case-by-case basis. These approximations might also involve giving up (i), and resorting again to make inference based on the informational content of (generally insufficient) summary statistics, an appropriate choice of which can also result in more robust estimation.

Bayesian methods are commonly employed for estimating DSGE models.[4] However, two features of DSGE models make Bayesian estimation simpler: (i) they produce analytical expressions for the behaviour of the agents around the steady state, and (ii) they involve only a limited number of different agents, hence equations (e.g. textbook-version NK models have just three equations). Having analytical expressions for the steady state behaviour allows (log-) linearisation[5] and the application of a simple Kalman filter to derive the likelihood and perform exact inference (on the approximated model). A limited number of equations implies that even if linearisation is not imposed, and a more complicated filter is used, simulation of the model is relatively fast. Still, the computational time required to repeatedly solve the model is an issue in DSGE modelling, as recognised by Fernández-Villaverde et al. (2016): "[C]losed-form solutions are the exception and typically not available for models used in serious empirical applications. [...] This ultimately leads to a trade-off: given a fixed amount of computational resources, the more time is spent on solving a model conditional on a particular θ, e.g., through the use of a sophisticated projection technique, the less often an estimation objective function can be evaluated. For this reason, much of the empirical work relies on first-order perturbation approximations of DSGE models, which can be obtained very quickly. The estimation of models solved with numerically sophisticated projection methods is relatively rare, because it requires a lot of computational resources."

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[2] As such, these techniques might be classified as advanced calibration. Kydland and Prescott (1996, p. 74) distinguish calibration from estimation by suggesting that calibration is concerned with data tracking (finding the values of the parameters that make the model behave as close as possible to the real data), while estimation refers to the inferential effort to learn about the underlying values of the parameters: "[D]ata are used to calibrate the model economy so that it mimics the world as close as possible along a limited, but clearly specified, number of dimensions. Note that calibration is not an attempt at assessing the size of something: it is not estimation. *Estimation* is the determination of the approximate quantity of something. Other authors, however, have a less clear-cut understanding of the two terms. For instance, Hansen and Heckman (1996, p. 91) say that " [T]he distinction drawn between calibrating and estimating the parameters of a model is artificial at best. Moreover, the justification for what is called calibration is vague and confusing. In a profession that is already too segmented, the construction of such artificial distinctions is counterproductive." This applies even more to a Bayesian approach to estimation, where no "true" values of the parameters are assumed to exist.

[3] The Bayesian estimator minimises the posterior expected loss with quadratic loss functions (mean squared error, MSE), where the expectation is taken over the posterior distributions of the parameters.

[4] See the recent review by Fernández-Villaverde et al. (2016).

[5] Linearisation however is not neutral: it eliminates asymmetries, threshold effects and many other interesting phenomena (Rubio-Ramirez and Fernández-Villaverde, 2005).