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Seismic motion control of structures: A developed adaptive backstepping approach

Fereidoun Amini *, Pedram Ghaderi

School of Civil Engineering, Iran University of Science and Technology, P.O. Box 16765-163, Narmak, Tehran, Iran

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ABSTRACT

This paper presents a robust control scheme to regulate seismic vibrations in MDOF structural systems. Due to difficulties in real-time measurement of ground motion acceleration and also intrinsic uncertain nature of structural damping, it is assumed that these data are not available for control purposes. First, these uncertain data are estimated by appropriate adaptive laws. Then a developed adaptive backstepping control strategy is used to propose a control law which only depends on feedback measurements of displacement and velocity vectors. Two numerical examples are provided and the result of the proposed approach is presented in comparison with LQR design.

1. Introduction

Control of dynamic response of structures subjected to natural phenomena such as strong winds and seismic ground motion is still one of the challenging topics for structural designers. This is essentially due to the uncertain nature of these types of loadings which makes it difficult to develop appropriate predictive models. On the other hand, due to technical deficiencies such as noise-induced measurements, time delay issues and economic issues, in many practical situations real-time measurement of the external disturbances (e.g., ground motion acceleration) is not feasible.

During the past decade, smart material technologies and invention of dampers and actuators with response time of the millisecond-order [1] have made many complex active control schemes practically possible. On the other hand, advances in real-time measurement of the properties and responses of the structures [2–4], help engineers to exploit these on-line data with more accuracy in identification of the system and to design more efficient active controllers. Consequently, nowadays adaptive controllers can be used in practice to deal with uncertain parameters such as undetermined damping or stiffness deficiency in structural systems. A brief literature review of this field in the last decade reveals promising experimental results [5,6] and efficient robust control methods [7,8] particularly designed for structural systems. A comprehensive review on advances in active structural control can be found in [9].

Backstepping is a recursive procedure that interlaces the choice of a Lyapunov function with the design of feedback control. It breaks a design problem for the full system into a sequence of design problems for lower order (even scalar) subsystems. By exploiting the extra flexibility that exists with lower order and scalar subsystems, backstepping can often solve stabilization, tracking, and robust control problems under conditions less restrictive than those encountered in other methods [10]. The methodology of backstepping design is comprehensively discussed in [11–13] by its pioneers.

There are good evidences that backstepping, like other Lyapunov-based methods, is a promising approach [14–16] for active control of structural systems. Particularly, in combination with adaptive laws, its ability to cope with a variety of uncertainties, in form of scalars, vectors and even matrices, makes it extremely interesting from a designer's point of view.

This paper studies an adaptive control strategy for a multi degrees of freedom (MDOF) structural system with saturation nonlinearity in control force and subjected to uncertain seismic ground motion. It is assumed that: (1) the seismic ground motion is unmeasured and hence unavailable for control purposes. (2) No material nonlinearity is involved (3) Stiffness and mass matrices are determined. (4) Damping matrix of the system assumed uncertain. (5) The system is in a reduced order realization with a moderate number of degrees of freedom (DOFs) where uncontrollable and unobservable DOFs are eliminated. Consequently, it is justified and reasonable to apply active control forces to all DOFs.

Based on these assumptions and using full state feedback, an adaptive backstepping design for the tracking error of displacement vector, with respect to a reference signal, is discussed. The capacity of active control actuators has considered as well. For this purpose we appropriately expanded the single-degree-of-freedom solution discussed in [17–18] to a MDOF uncertain structural system. The proposed approach, performed in two simple steps, does not require decomposing the system equations from matrix into scalar arithmetic which results in a well-organized closed-form solution for the active control force vector. As a practical example

* Corresponding author. Tel.: +98 21 77240332; fax: +98 21 77240398.
E-mail addresses: famini@iust.ac.ir (F. Amini), p_ghaderi@iust.ac.ir (P. Ghaderi).

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the scheme presented here can be used in control of adjoining buildings connected at all levels with active actuators. In this scenario the designer uses the stronger building as a support for the weaker or damaged one.

The rest of the paper is organized as follows: Section 2 summarizes the notations and assumptions we made through this paper. Section 3 is the main part of the paper which presents the backstepping design for a MDOF system. In this section the main steps of the proposed method are stated in the form of a theorem. The proofs are given in Appendix A. Application of the backstepping controller is then presented in Section 4 where the results of the proposed method are compared with a common linear quadratic regulator (LQR) design. Conclusion remarks and a brief discussion are presented in Section 5.

2. Preliminaries

2.1. Notations

Throughout this paper the matrices are defined with capital bolded letters, vectors with small bolded letters and scalars with italic letters. The element-wise sign function is as follows:

$$\mathrm{sgn}(\mathbf{x}) := [sgn(x_1), \ldots, \mathrm{sgn}(x_n)]^T : \mathrm{sgn}(x_i) = \begin{cases} -1 & : \quad x_i < 0 \\ 0 & : \quad x_i = 0 \\ +1 & : \quad x_i > 0 \end{cases} \tag{1}$$

Additionally, considering the column vector $\{\mathbf{x}\}_{n\times 1} = [x_1, \ldots, x_n]^T$, $|\mathbf{x}| = [|x_1|, \ldots, |x_n|]^T$ denotes the element-wise absolute value function, $\|\mathbf{x}\|_\infty = \max_j |x_j|$ the infinity norm, $\|\mathbf{x}\|_2^2 = \mathbf{x}^T\mathbf{x}$ the Euclidean norm, $\|\mathbf{x}\|_1 = \mathbf{x}^T \mathrm{sgn}(\mathbf{x})$ the L_1 norm.

In practice, depending on the type and model of each actuator, the produced force is limited to a maximum amount. In mathematical modeling, a common method to take into account this constraint is through the saturation function which is defined in Eq. (2). Here $\boldsymbol{f}$ denotes the control force vector and $F_{\max,i}$ is the saturation index for the ith element of $\mathbf{f}$ which indicates the maximum feasible force of the corresponding actuator.

$$sat(\{\mathbf{f}\}_{n\times 1}) := \{sat(f_1), \ldots, sat(f_n)\}^T : \quad sat(f_i) = \begin{cases} f_i & |f_i| < F_{\max,i} \\ \mathrm{sgn}(f_i) \times F_{\max,i} & |f_i| \geqslant F_{\max,i} \end{cases} \tag{2}$$

2.2. Assumptions

The dynamics of a structural system, subjected to seismic ground motion, can be described by a second order matrix differential equation usually composed of a large number of degrees of freedom. Concerning these large scale systems one may encounter in practical control design problems, it is a common technique to first derive a simplified model as an approximation of the complex one and then use that for control design purposes. In this context complexity is measured by the number of system states. Roughly speaking, the so-called reduced order model can be derived from the finite element model keeping only a set of controllable DOFs and those selected by the control designer.

Assumption #1. For the current study we assume that a reduced-order realization of the structural system is derived and available in advance of the control design procedure.

The system equations are presented below. It should be noted that the capacity of each actuator is considered as well and, as mentioned previously, this issue has entered the formulation via saturation function:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{3}$$

$$\dot{\mathbf{x}}_2 = -\mathbf{M}^{-1}\mathbf{D}\mathbf{x}_2 - \mathbf{M}^{-1}\mathbf{K}\mathbf{x}_1 - \mathbf{l_a}a_g + \mathbf{M}^{-1}sat(\mathbf{f}) \tag{4}$$

In the above equations, denoting the number of DOFs by n, $\mathbf{x}_1 \in \mathbf{R}^n$ represents the displacement vector. $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{D} \in \mathbf{R}^{n\times n}$ are the mass, stiffness and damping matrices. a_g is the time-dependent ground motion acceleration. The zero-one column vector, $\mathbf{l_a}$, is used to allocate a_g to the respective DOFs. $\{sat(\mathbf{f})\}_{n\times 1}$ denotes the active control force filtered by saturation function. Note that as Eq. (2) shows clearly, saturation thresholds can be selected differently. These values are selected based on the capacity of actuators installed in the structural system.

Assumption #2. The active control forces are applied to all DOFs of the system, i.e., $\{\mathbf{f}\}_{n\times 1}$ has n element. This assumption is justified by the fact that, instead of the complex finite element model, a reduced-order model will be used for the control design.

Assumption #3. The ground motion acceleration, a_g, is unavailable for control purposes. On the other hand, system's states $\mathbf{x}_1$ and $\mathbf{x}_2$ (displacement and velocity vectors) are measured in real-time and available.

Assumption #4. No material nonlinearity is involved. The stiffness and mass matrices are constant and known.

Assumption #5. Rayleigh damping is used to model internal structural damping. This is illustrated in Eq. (5). The values of scalar coefficients $\mu_1, \mu_2 \in \mathbf{R}^+$ are uncertain and not available for control purpose, i.e., the damping matrix cannot explicitly participate in control force signal.

$$\mathbf{D} = \mu_1\mathbf{M} + \mu_2\mathbf{K} \tag{5}$$

It should be noted that the nature of uncertainty in ground motion acceleration is different from the one in damping matrix. This is due to the fact that the former is a time-dependent signal and the latter depends on two undermined constant coefficients.

Assumption #6. The structural system is bounded-input bounded-output stable. i.e., for ground accelerations of finite magnitude the response of the system cannot grow to infinity which is an evident engineering assumption.

3. Active control scheme using developed adaptive backstepping

It is assumed that the designer intends to force the displacement vector of the structural system to follow a known constant or time-dependent reference signal. Clearly, the control scheme one may suggest should be robust against external disturbance (e.g., seismic motion) as well. The simplest case for the reference signal is constant zero, i.e., from the beginning of the control the structural systems will be forced to directly reach the static configuration. However, thanks to the flexibilities of the backstepping method, which is the subject of this paper, other scenarios are possible as well. For example the designer may intend to mitigate the effects of seismic loadings on a common shear building by forcing it to follow a damped harmonic movement along the second natural mode shape of the structure.

Before we move to the mathematical representation, it is suitable to state the main idea we follow: First, to estimate the uncertain data (damping matrix and ground acceleration) by proper adaptive laws. Second, to calculate a proper active control force based on state feedbacks.

More precisely, considering the reduced-order system Eqs (3) and (4), the control objectives are: (1) To design the control force, $\{\mathbf{f}\}_{n\times 1}$, only using mass and stiffness matrices along with system's states, $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$. (2) Denoting the reference signal by the vector $\mathbf{x}_r$, we also want to keep the tracking error, $\mathbf{x}_1 - \mathbf{x}_r$, adjustable by choice of design parameters.