Journal of Economic Dynamics & Control 73 (2016) 453–457

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Dynamic R&D with spillovers: A comment

Grega Smrkolj [a,*], Florian Wagener [b]

[a] Newcastle University Business School, 5 Barrack Road, Newcastle upon Tyne NE1 4SE, UK
[b] CeNDEF, Amsterdam School of Economics, Universiteit van Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands

ARTICLE INFO

Article history:
Received 12 April 2016
Received in revised form
13 October 2016
Accepted 30 October 2016
Available online 4 November 2016

JEL classification:
C73
D43
L13
O31

Keywords:
Differential games
Process innovation
Spillovers

ABSTRACT

Cellini and Lambertini [2009. Dynamic R&D with spillovers: competition vs cooperation. J. Econ. Dyn. Control 33, 568–582] study a dynamic R&D game with spillovers. This comment demonstrates that, contrary to what is claimed in their paper, the game is not state redundant and the open-loop Nash equilibrium is not subgame perfect.

1. Introduction

Dynamic models are popular in modern industrial organization. They allow to model firms smoothing their investments over a long time, as well as reacting to each other's past actions. Cellini and Lambertini (2009), CL in what follows, presented a continuous-time generalization of the seminal static R&D model of d'Aspremont and Jacquemin (1988). Their paper compares R&D incentives of firms that compete on R&D to those that cooperate on R&D. They claim that in a dynamic model the conflict between individual and social incentives does not necessarily arise, unlike the situation for the static model.

In particular, their analysis consists of three steps: they characterize the open-loop Nash equilibrium, they claim to prove that it is subgame perfect, and then they analyze the steady-state allocation. Their proof of subgame perfectness rests on the claim that the closed-loop equilibrium collapses to the open-loop equilibrium.

The aim of this comment is to show that the second step of their analysis, embodied in their Lemma 1, is incorrect. First we shall show that the proof of this lemma is flawed, and subsequently we give a simple argument why the statement of the lemma cannot hold either. That is, we show that the open-loop equilibrium is not subgame perfect and the game is not state redundant or perfect. The solution analyzed in their paper therefore reduces to the open-loop situation, where firms commit to the entire investment schedule at the beginning of the game.

* Corresponding author.
E-mail addresses: grega.smrkolj@newcastle.ac.uk (G. Smrkolj), f.o.o.wagener@uva.nl (F. Wagener).

http://dx.doi.org/10.1016/j.jedc.2016.10.011
0165-1889/

2. The model

We quickly summarize the model of CL. Time $t \geq 0$ is continuous. There are two firms that compete in a market with market demand

$$p(t) = A - q_1(t) - q_2(t). \tag{1}$$

Firms decide simultaneously how much to produce (q_i) and how much R&D effort to exert (k_i). Instantaneous production costs are $C_i(t) = c_i(t)q_i(t)$, $i = 1,2$, where c_i is the marginal cost of firm i. Marginal costs evolve over time as

$$\dot{c}_i(t) = c_i(t)\big(-k_i(t) - \beta k_j(t) + \delta\big), \tag{2}$$

where, as always, $j \neq i$, where $0 \leq \beta \leq 1$ is the level of spillover, and where $\delta \geq 0$ is the technology depreciation rate. R&D costs (Γ_i) are quadratic,

$$\Gamma_i(k_i) = bk_i^2, \tag{3}$$

with $b > 0$, and the instantaneous profit of firm i is therefore

$$\pi_i(t) = \big(A - q_i(t) - q_j(t) - c_i(t)\big)q_i(t) - bk_i(t)^2. \tag{4}$$

Total discounted profits are

$$\Pi_i = \int_0^\infty \pi_i(t)e^{-\rho t}dt, \tag{5}$$

where $\rho > 0$ is a constant discount rate that is equal for both firms. The optimal control problem of firm i is to find controls q_i^* and k_i^* that maximize the profit functional Π_i subject to the state equations (2) and the initial conditions $c_i(0) = c_{i0}$.

3. The open-loop Nash equilibrium is not subgame perfect

3.1. Subgame perfectness and time consistency

Introduce the notation $u_i(t)$ for an open-loop strategy. Recall that an open-loop Nash equilibrium $(u_1^*(t), u_2^*(t))$ of this differential game is *subgame perfect*, or *strongly time consistent*, if for every time $T > 0$, we can change the strategies $u_i^*(t)$ for times $0 \leq t < T$ at will, as long as the resulting strategies are still admissible, and the resulting strategies still provide an open-loop Nash equilibrium for $t \geq T$.

The equilibrium is *time consistent*, or *weakly time consistent*, if after having played up to time T according to the strategies $u_i^*(t)$, the players are given the option to reconsider their strategies for the remainder of the time, and the strategies $(u_1^*(t), u_2^*(t))$, restricted to $t \geq T$, still form an open-loop Nash equilibrium.

3.2. First argument

CL claim, in their Lemma 1, that the open-loop equilibrium of this game is subgame perfect.

We contest this. Our argument runs as follows: Fershtman (1987) showed that to be subgame perfect, a Nash equilibrium in open-loop strategies has to be an equilibrium in feedback strategies; in particular the open-loop equilibrium strategies have to be independent of initial conditions. For infinite horizon games like the present one, where the only explicit time dependence is exponential discounting, the set of feedback strategies is necessarily invariant under time-shifts (Basar and Olsder, 1999). That is, if $t \mapsto u^*(t)$ is an equilibrium strategy tuple, then so is $t \mapsto u^*(\tau + t)$, for each $\tau > 0$. Moreover, if $u^*(t)$ tends to a limit u_∞^* as $t \to \infty$, which is the case in the present game, then $u(t) = u_\infty^*$ is also an equilibrium feedback strategy tuple, which is moreover independent of both time and initial conditions; that is, it is a real constant. This follows from letting τ tend to infinity.

In the present game, this would imply that the state variables are constant — as a consequence of Eq. (16) below — and hence that every state is a steady state. However, CL have showed that under open-loop Nash equilibrium dynamics, there are at most three steady states. This constitutes a contradiction.

A second, more detailed argument is given in Section 3.5.

3.3. First-order optimality conditions

It follows that the proof of Lemma 1 cannot be correct: we shall try to point out its flaws.

CL use the memoryless closed-loop information structure (cf. Basar and Olsder, 1999): a strategy is memoryless closed-loop, if it conditions the action of the player on the current state and time, as well as on the initial state. That is, in the