Journal of Economic Dynamics & Control 66 (2016) 1–19

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Optimal asset allocation with fixed-term securities[☆]

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ARTICLE INFO

Article history:
Received 27 March 2015
Received in revised form
27 February 2016
Accepted 1 March 2016
Available online 7 March 2016

JEL classification:
G11
D14

Keywords:
Optimal portfolio
Fixed-term investment
Bank deposit
Closed-end security
Martingale method

ABSTRACT

We investigate the optimal asset allocation of an investor who can invest in a fixed-term security that is only traded at time 0. Using a generalized martingale approach, we solve the investor's optimal portfolio problem, determine the optimal allocation to fixed-term securities, and provide a representation of trading strategies in terms of a liquidity-related derivative. We apply our approach to two benchmark scenarios: fixed-term fixed-rate bank deposits, and unspanned closed-end securities that can only be traded at time 0. We show that both can be key parts of the investor's optimal asset mix, and we investigate the dependence of optimal allocations to fixed-term investments, implied liquidity premia and other characteristics on the underlying model parameters.

1. Introduction

Fixed-term investments constitute an integral part of private investors' financial portfolios. According to the Federal Reserve Statistical Release (2015), time and savings deposits alone make up more than 11% of U.S. households' total financial assets ($8.0 trillion), and equity in non-corporate businesses amounts to 15% of household wealth ($10.4 trillion). Liquid assets such as corporate equities and shares in mutual funds represent 19% and 12% of total financial assets, while checkable deposits and currency amount to less than 2%.[1] While the latter are liquidly traded, the common feature of fixed-term investments such as bank deposits or non-tradable stock is that they are not easily converted into currency. As a compensation for this reduction in liquidity, such securities typically offer excess returns over their liquid counterparts. An investor that has access to both liquid and fixed-term investment opportunities therefore faces a non-trivial, two-step allocation problem:

(1) How much capital should be invested fixed-term?

[☆] Previous versions circulated under the title “Optimal Investment with Illiquid Assets”.

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[1] More generally, the importance of illiquid assets for individuals' overall (not necessarily financial) wealth is also well-documented in a literature that is too large to review here. We refer to the overview of Ang et al. (2014) and the references therein, and only mention here that individuals typically hold more than 80% of their total net worth in illiquid positions, with housing wealth making up about 50% ($25.0 trillion), see Iacovello (2011).

http://dx.doi.org/10.1016/j.jedc.2016.03.001
0165-1889/

(2) How should the remaining wealth be invested in liquid securities?

In this paper, we therefore investigate a stylized version of this portfolio problem where the investor that has the opportunity to invest in

(a) a set of standard, perfectly liquid assets, and
(b) a fixed-term security that can only be traded at the beginning of the investment period.

We demonstrate that the numbers above appear quite plausible in realistic parametrizations of our model, thus offering a possible explanation for the predominance of fixed-term securities in private investors' financial portfolios. For instance, a fixed-term deposit rate of 0.5% in excess of the riskless rate suffices to induce a real-world investor with a 5 year time horizon to allocate 60% of her financial wealth to bank deposits.

One key feature of fixed-term securities is their illiquidity. There is a large literature on investment with illiquid assets, emphasizing a variety of different aspects of illiquidity. Illiquidity effects can be generated by transaction costs or quantity limitations as in Constantinides (1986), Grossman and Laroque (1990), Longstaff (2001) and Lo et al. (2004), or via differential borrowing costs as in Cvitanić and Karatzas (1992), Korn (1995) and Gârleanu (2009). In these approaches, a certain degree of liquidity can be achieved, albeit at a cost. An alternative approach is to assume that illiquidity risk means that some asset markets are closed for certain time intervals of possibly random duration. This includes both recent research on portfolio optimization with stochastic endowments and solvency constraints, see Cuoco (1997), Munk (2000), Hugonnier and Kramkov (2004) and Hugonnier et al. (2005), and Huang (2003), Diesinger et al. (2010) and others, where illiquidity is modeled via exponential blackout periods. The approach adopted in this paper is closest to that of Kahl et al. (2003), Longstaff (2009), Dai et al. (2015) and others, where liquidity is modeled as a deterministic, one-shot event. A common feature of these articles is that the investor's position in illiquid assets, or the stochastic endowment, is exogenously given.

This paper contributes an analysis of investors' endogenous allocation to fixed-term investments, as well as a new methodological approach inspired by the abstract duality results of Hugonnier and Kramkov (2004). In particular, we investigate in detail the comparative statics of fixed-term investments in private investor's optimal portfolio choices. Our findings complement and qualify those of the literature on optimal investment with illiquidity in general to the case of fixed-term securities: both bank deposits and fixed-term securities are important for an optimal asset mix (see Ang et al., 2014 and others); bank deposits are particularly attractive for risk averse agents, while the desirability of risky fixed-term securities depends crucially on their correlation with spanned instruments (compare, e.g., Kahl et al., 2003); fixed-term securities of either type are particularly attractive for agents with short investment horizons (see, among others, Longstaff, 2009); and liquid market trading may have to be modified significantly to take into account fixed-term holdings (compare, e.g., Dai et al., 2015). Moreover, the subjective values attributed to fixed-term investment opportunities are significant, and implied liquidity premia of closed-end securities vary between 5% and 15% for realistic parameter values.

Moreover, we wish to stress that the simplified view of liquidity taken in this paper is suitable for a number of securities including bank deposits, closed-end funds and other fixed-term investments where it is in the nature of the security that it cannot be traded for a certain time duration. However, it is not a methodology for portfolio optimization with illiquid securities in general. In particular, our approach does not apply to asset allocation problems with infrequently traded assets, different degrees of liquidity, or labor income.

Organization of the paper. Section 2 formalizes the optimal investment problem including fixed-term securities. In Section 3 we solve the portfolio problem using a generalized martingale approach. Section 4 characterizes the investor's optimal trading strategy in liquid assets in terms of a liquidity-related derivative. The theoretical results are illustrated in two benchmark case studies in Section 5: fixed-term bank deposits and unspanned closed-end funds. Section 6 concludes.

2. Optimal investment with fixed-term securities

We set up the portfolio problem for an investor in a financial market that includes, in addition to a set of standard, perfectly liquid assets, a fixed-term investment opportunity. In contrast to the liquidly traded assets, this security can only be traded at the beginning of the investment period.

Liquid securities market. We consider a probability space $(\Omega, \mathfrak{A}, \mathbb{P})$ equipped with a filtration $\mathfrak{F}$ satisfying the usual conditions, and we denote by $T>0$ the investor's time horizon. $\mathfrak{F}_t$ represents the information available to market participants at time $t \in [0,T]$. Liquid assets are assumed to follow standard dynamics: the money market account B satisfies

$$dB_t = B_t r_t \, \mathrm{d}t$$

with an $\mathfrak{F}$-progressively measurable short rate process r, and the cum-dividend price of the risky asset P, a stock or stock index, follows the dynamics

$$dP_t = P_t[(r_t+\eta_t)\, \mathrm{d}t + \sigma_t \, \mathrm{d}W_t]$$

with $\mathfrak{F}$-progressively measurable excess return and volatility processes η and σ, and an $(\mathfrak{F}, \mathbb{P})$-Wiener process W. We suppose that $\mathfrak{F}$ is generated by W and the class of $\mathbb{P}$-null sets. Thus the market of liquidly traded securities is $\mathfrak{F}_T$-complete,