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A novel evidence-theory-based reliability analysis method for structures with epistemic uncertainty

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ABSTRACT

Evidence theory has a strong ability to deal with the epistemic uncertainty, based on which the uncertain parameters existing in many complex engineering problems with limited information can be conveniently treated. However, the large computational cost caused by its discrete property severely influences the practicability of evidence theory. This paper aims to develop an efficient method to evaluate the reliability for structures with epistemic uncertainty, and hence improve the applicability of evidence theory in engineering problems. A uniformity approach is used to deal with the evidence variables, through which the original reliability problem can be transformed to a traditional reliability problem with only random uncertainty. It is then solved by using a response-surface-based reliability analysis method, and a *most probable point* (MPP) is obtained. Based on the MPP, the most critical focal element which has the maximum contribution to failure can be identified. Then using an approximate model created based on this focal element, the reliability interval can be efficiently computed for the original epistemic uncertainty problem. Three numerical examples are investigated to demonstrate the effectiveness of the present method, which include two simple problems with explicit expressions and one engineering application.

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1. Introduction

Products should be reliable, robust and safe against uncertainty such as manufacturing imprecision, usage variation and imperfect knowledge, so quantifying, controlling and managing the effects of uncertainty at the design stage is important, sometimes even imperative [1]. According to the idea that uncertainty is viewed as the difference between the present state of knowledge and the complete knowledge, it can be classified into aleatory and epistemic types [2]. *Aleatory uncertainty*, also named as objective or stochastic uncertainty, describes the inherent variation associated with a physical system or environment. The probability theory is widely used to deal with the aleatory uncertainty when sufficient information is available to estimate precise probability distributions. So far many probability-based reliability analysis techniques have been well established and successfully applied to varieties of industrial fields [3–6]. On the other hand, *epistemic uncertainty* is referred to as the lack of knowledge or information in some phases or activities of the modeling process [7], and hence the collection of more information or an increase of knowledge would help decrease the level of uncertainty. Different kinds of theories have been developed to handle the epistemic uncertainty, which include possibility theory [8–10], fuzzy sets [11], convex models [12–19] and evidence theory (or Dempster–Shafer theory) [20–24]. In possibility theory, evidence from different experts is always consistent [10]. Fuzzy sets utilize the membership functions to characterize the input uncertainty [11]. For many complex problems only a variation bound of the uncertainty can be obtained, and in this case convex models have been extensively applied to characterize and propagate input uncertainty in order to calculate the interval of the uncertain output [21]. Also, some theories have been developed to deal with the aleatory and epistemic uncertainty simultaneously, which include the p-box approach [25,26] and fuzzy probabilities [27,28]. P-box approach is specified by the left and right bounds on the cumulative probability distribution function [25]. Fuzzy probabilities utilize fuzzy random variables to extend reliability analysis to situations when the outcomes of some random experiment are fuzzy sets [27].

Evidence theory seems to be more general than other uncertainty modeling techniques [30]. Under different cases, evidence theory will be equivalent to the classical probability theory, possibility theory, p-box approach, fuzzy sets and convex models, respectively. It can deal with limited and even conflicting information from experts. Furthermore, the basic axioms of evidence theory allow us to combine aleatory and epistemic uncertainty in a straightforward way without any assumptions [1]. Due to the above advantages, evidence theory has been extended into the

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structural reliability analysis in recent years, and some exploratory work in this area has been reported. Evidence theory was applied to deal with the uncertainty in rock engineering, based on which a reliability-based design of tunnels was accomplished [29]. The use of evidence theory in engineering reliability was investigated through a simple algebraic function, and the strengths and weaknesses of evidence theory were also concluded [30]. Evidence theory and Bayesian approaches were compared in uncertainty modeling and decision making under epistemic uncertainty [31]. A reliability-based optimization algorithm was constructed based on evidence theory for multidisciplinary design optimization (MDO) [32]. The use of several uncertainty modeling techniques (i.e. probability model, evidence theory, possibility theory, and interval analysis) was explored and compared through some benchmarks, and hence a unified framework was provided for uncertainty propagation analysis of these different methods [33]. Through creating a multi-point approximation at a certain point on the limit-state surface, a reliability analysis method was proposed for structures with epistemic uncertainty [34,35], which has made an important contribution to improve its computational efficiency. Three selected metamodeling techniques for reliability analysis using evidence theory are compared through six numerical examples [36]. A reliability-based design optimization (RBDO) using evidence theory was developed based on a gradient projection technique [37]. A semi-analytic approach [38] and a sampling-based approach [39] were developed for sensitivity analysis of the uncertainty propagation problems using evidence theory. An evidence-based design optimization (EBDO) method was proposed, which can quickly identify the vicinity of the optimal point and also the active constraints [40]. By combining evidence theory and conventional probability model, a reliability analysis method was proposed for epistemic and aleatory mixed uncertainties [41]. A RBDO method was also formulated based on evidence theory and interval analysis technique, and it was applied to the design of a pressure vessel [42].

Though some important progresses have been made in the above work, presently evidence theory has been barely used in complex engineering problems. One main reason is its high computational cost, as indicated in [35]. Unlike the probability density function (PDF) in probability model and the membership function in fuzzy sets, the uncertainty is propagated through a discrete basic probability assignment by using evidence theory, which is generally described by a series of discontinuous intervals rather than an explicit function. Thus the expensive computational cost might be inevitable for a multidimensional problem when using the evidence theory to conduct reliability analysis. So far, some numerical methods [34–36] have been developed to improve the computational efficiency. For these methods, a sampling center on the limit-state function is firstly obtained by an optimization algorithm and based on which a response surface is constructed to calculate the reliability interval. Thus, in this paper, we refer to this kind of methods as the response surface reliability analysis method based on a sampling center from optimization (RSRO). However, it seems not always an easy job to provide satisfied reliability analysis results using the RSRO method. Because the reliability analysis accuracy of this method is significantly influenced by the basic probability assignment for each parameter. Therefore, to improve the applicability of evidence theory, it seems necessary to develop some new reliability analysis methods with high efficiency, fine robustness and also strong adaptibility.

This paper aims to develop a new reliability analysis method for structures with epistemic uncertainty, in which a concept of most probable focal element is proposed and based on it the computational cost of reliability analysis can be significantly reduced. The remainder of this paper is organized as follows. Some basic conceptions for evidence theory are introduced in Section 2. A reliability analysis procedure using evidence theory is given in Section 3. An efficient algorithm is formulated to compute the reliability interval in Section 4. Three numerical examples are investigated in Section 5 and some conclusions are finally summarized in Section 6.

2. Basic conceptions of evidence theory

Evidence theory was proposed by Dempster and Shafer [20] and its main concept is that our knowledge on a given problem can be inherently imprecise. Thus, an interval composed of the *belief measure* (Bel) and the *plausibility measure* (Pl) is employed to characterize the uncertainty of the system response.

Evidence theory starts by defining a *frame of discernment* (FD) that is a set of mutually exclusive elementary propositions and it can be viewed as a finite sample space in probability theory. For instance [35], if FD is given as $X = \{x_1, x_2\}$, then we will have two mutually exclusive elementary propositions x_1 and x_2. All the possible subset propositions of X will form a power set 2^X, and for the above example it has $2^X = \{\emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$.

As an important concept in evidence theory, the *basic probability assignment* (BPA) represents the degree of belief for a proposition. The BPA is assigned through a mapping function $m : 2^X \to [0, 1]$, which should satisfy the following three axioms:

$$m(A) \geqslant 0 \quad \text{for any } A \in 2^X \tag{1}$$

$$m(\emptyset) = 0 \tag{2}$$

$$\sum_{A \in 2^X} m(A) = 1 \tag{3}$$

where $m(A)$ refers to the BPA corresponding to the event A and it characterizes the amount of "likelihood" that can be assigned to A but to no proper subset of A.

Sometimes the available evidence may come from different sources, and such bodies of evidence can be aggregated using existing rules of combination. Dempster' rule is the most popular rule for combination, and for two BPAs m_1 and m_2 the combined evidence can be calculated as follows [20]:

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K} \quad \text{for } A \neq 0 \tag{4}$$

where

$$K = \sum_{B \cap C = 0} m_1(B) m_2(C) \tag{5}$$

K represents the total conflict between the two independent sources or experts, hence Dempster's rule is generally appropriate for evidence with relatively small amounts of conflict. When there is severe or total contradiction among the evidence from different sources, some modified versions [22,24] of Dempster's rule can be used.

For the lack of information, generally it cannot provide a deterministic value for a proposition A as in the probability theory, but only an interval $[\mathrm{Bel}(A), \mathrm{Pl}(A)]$. The lower bound $0 \leqslant \mathrm{Bel}(A) \leqslant 1$ and the upper bound $0 \leqslant \mathrm{Pl}(A) \leqslant 1$ are called the belief measure and the plausibility measure, respectively, and they are defined as below:

$$\mathrm{Bel}(A) = \sum_{C \subseteq A} m(C) \tag{6}$$

$$\mathrm{Pl}(A) = \sum_{C \cap A \neq \emptyset} m(C) \tag{7}$$

where $\mathrm{Bel}(A)$ is the summary of all the BPAs of propositions that totally agree with the proposition A, and $\mathrm{Pl}(A)$ is the summary of all

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