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The stability of macroeconomic systems with Bayesian learners

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ABSTRACT

We study abstract macroeconomic systems in which expectations play an important role. Consistent with the recent literature on recursive learning and expectations, we replace the agents in the economy with econometricians. Unlike the recursive learning literature, however, the econometricians in the analysis here are Bayesian learners. We are interested in the extent to which expectational stability remains the key concept in the Bayesian environment. We isolate conditions under which versions of expectational stability conditions govern the stability of these systems just as in the standard case of recursive learning. We conclude that Bayesian learning schemes, while they are more sophisticated, do not alter the essential expectational stability findings in the literature.

1. Introduction

1.1. Overview

A large and expanding literature has developed over the last two decades concerning the issue of learning in macroeconomic systems. These systems have a recursive feature, whereby expectations affect states, and states feed back into the expectations formation process being used by the agents. The focus of the literature has been on whether processes in this class are locally convergent to rational expectations equilibria. Evans and Honkapohja (2001), in particular, have stressed that the expectational stability condition governs the stability of real-time learning systems defined in this way.

This line of research has so far emphasized recursive updating, including least squares learning as a special case. There has been little study of Bayesian updating in the context of expectational stability. What might one expect from an extension to Bayesian updating? There seem to be at least two lines of thought in this area. One is that Bayesian estimation is a close relative of least squares, and therefore that all expectational stability results should obtain with suitable adjustments, but without conceptual difficulties. A second, opposite view is that Bayesian agents are essentially endowed with rational expectations—indeed Bayesian learning is sometimes called "rational learning" in the literature—and therefore one should not expect to find a concept of "expectational instability" in the Bayesian case. A goal of this paper is to understand which of these views is closer to reality in abstract macroeconomic systems.

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It is also important to understand how Bayesian updating might repair certain apparent inconsistencies in the recursive learning literature. Cogley and Sargent (2008), for example, have noted that there are "two minds" embedded in the anticipated utility approach to learning that has become popular. According to Cogley and Sargent (2008, p. 186),

> "[The anticipated utility approach recommended by Kreps (1998)] is of two minds …. Parameters are treated as random variables when agents learn but as constants when they formulate decisions. Looking backward, agents can see how their beliefs have evolved in the past, but looking forward they act as if future beliefs will remain unchanged forever. Agents are eager to learn at the beginning of each period, but their decisions reflect a pretence that this is the last time they will update their beliefs, a pretence that is falsified at the beginning of every subsequent period."

In this paper, we take a first step toward studying these issues in the context of expectational stability. The Bayesian econometricians in our model will recognize that their beliefs will continue to evolve in the future. The Bayesian perspective means treating estimates as random variables, and is one way to take parameter uncertainty into account. We think of this as a step toward rationality. However, the Bayesian econometricians in our model may still be viewed as boundedly rational rather than fully rational, because, while agents acknowledge that their beliefs will change over time they do not have an explicit model of this process. That is, they have a Bayesian perspective but they do not model their own learning behavior. In this sense we see our paper as one step closer toward rationality relative to the classical recursive learning case.

1.2. What we do

We consider a standard version of an abstract macroeconomic model, the generalized linear model of Evans and Honkapohja (2001). Instead of assuming standard recursive learning, we think of the private sector agents as being Bayesian econometricians. In particular, the agents will then treat estimated parameters as random variables. In certain circumstances, the system will behave as if the agents are classical recursive learners, but in general, the system will behave somewhat differently from the one where agents are classical econometricians. We highlight these differences and similarities. The primary question we wish to address is whether we can describe local convergence properties of systems with Bayesian learners in the same expectational stability terms as systems with standard recursive learning.

1.3. Main findings

We find expectational stability conditions for systems with Bayesian learners. We are able to isolate cases where these conditions are identical to the conditions for non-Bayesian systems. In these cases, in terms of expectational stability, the Bayesian systems yield no difference in results *vis-a-vis* the systems with standard recursive learning.

The actual stochastic dynamical systems produced by the classical recursive learning versus the Bayesian learning assumptions are not identical, however, except under special circumstances. This means that the dynamics of the two systems will differ during the transition to the rational expectations equilibrium, even if the local asymptotic stability properties do not differ. We document via examples how the dynamics of Bayesian systems can differ from the dynamics of non-Bayesian systems with identical shock sequences. We show situations in which the differences can be material and situations where the differences are likely to be negligible.

We interpret these findings as follows. When we replace the rational expectations agents in a model with recursive least squares learners, as has been standard in this literature, we are assuming a certain degree of bounded rationality. This has been discussed extensively in the literature. However, since the systems can converge, locally, to rational expectations equilibrium, the bounded rationality eventually dissipates, which is perhaps a comforting way to think about how rational expectations equilibrium is actually achieved. Still, one might worry that if the agents were a little more rational at the time that they adopt their learning algorithm, the local stability properties of the rational expectations equilibrium might be altered. Here, "a little more rational" means that the agents use Bayesian methods while learning instead of classical recursive algorithms, and so they take into account that they will be learning in the future. It is conceivable that equilibria which were unstable under standard recursive learning might now be stable under Bayesian learning, for instance. The results in this paper suggest that this is not the case. The expectational stability conditions for the systems with Bayesian learners are not any different, at least in the cases analyzed here, from those which are commonly studied in the literature. This suggests that the stability analysis following the tradition of Marcet and Sargent (1989) and Evans and Honkapohja (2001) may have very broad appeal, and that the assumption of standard recursive learning may be less restrictive than commonly believed.

1.4. Recent related literature

Bray and Savin (1986) studied learning in a cobweb model and noted that a recursive least squares specification for the learning rule implied that agents assumed fixed coefficients in an environment where coefficients were actually time-varying.[1] They thought of this as a misspecification, a form of bounded rationality. They asked whether convergence to

[1] This is the same concern raised by Cogley and Sargent (2008).