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A new approach to risk-return trade-off dynamics via decomposition

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ABSTRACT

This paper revisits the puzzling time series relation between risk premium and conditional volatility by proposing a flexible risk-return trade-off that allows for a variety of possible shapes and incorporates potential nonlinearities inherent in excess return dynamics. We derive this flexible risk-return relation using the decomposition approach of Anatolyev and Gospodinov (2010), which splits excess returns into the product of absolute returns and signs. Using this decomposition strategy, we study four major international financial markets. The empirical results support a significant and positive risk-return trade-off that is driven by conditional volatility, market timing and the interdependence between the two components, which is generically related to return skewness.

1. Introduction

Risk aversion is a fundamental concept in economics and finance and states that, all else equal, investors require additional reward for bearing additional risk. Such a relationship can be measured in a general form by the partial risk-return relation $\partial E(R)/\partial V$, where $E(R)$ is the expected reward and V is a proxy for risk. For a risk-averse investor, a positive risk-return relation, $\partial E(R)/\partial V > 0$, is expected.[1] Asset pricing theory implies that the risk-return relation, which depends on cash flow uncertainty and investors' utility functions, at least in a general equilibrium framework, can be complex and nonlinear (Whitelaw (1994, 2000)). For example, in the context of a simple endowment economy, Backus and Gregory (1993) show that the relation between expected return and risk can take nearly any shape depending upon investors'

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[1] In advanced portfolio theory, different kinds of risk are taken into consideration to measure the fluctuations of financial returns, such as L_1 distance (absolute return, high-minus-low), and L_2 distance (variance, standard deviation). However, if the theory of risk aversion is true, we should expect a positive and significant risk-return relation regardless of measurable functions of risk.

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preferences and the stochastic nature of an economy. Alternatively, under restrictive assumptions, Merton (1973) derives a simplified partial risk-return relation between expected return and variance that is linear and time invariant. Subsequently, a large body of academic research has estimated and tested this linear relation.

Unfortunately, empirical evidence on the linear risk-return trade-off, i.e., the mean-variance trade-off, is mixed and inconclusive. Some evidence supports a positive risk-return trade-off,[2] other evidence suggests a negative relation,[3] and a third strand of the literature finds that the relation is unstable and varies through time.[4]

Due to the mixed and inconclusive result for the linear risk-return trade-off, a number of recent studies have retrospected back to the general partial risk-return relation without imposing strong modeling assumptions.[5] Rossi and Timmermann (2012) argue that there is no theoretical reason for assuming a linear relationship between expected market return and market risk, and using an approach based on boosted regression trees, the authors find strong empirical evidence for a non-monotonic risk-return relation. In addition, Ghysels et al. (2014) use a MIDAS model for absolute returns that allows for possible switches in the risk-return relation through a Markov-switching specification, and subsequently the authors find evidence for regime changes in the risk-return relation. Further, Salvador et al. (2014) and Wu and Lee (2015) report the distinct reactions of the risk-return relation to high- and low-volatility periods and bull and bear markets, respectively. Their findings suggest that the risk-return relation responds asymmetrically to changes in states of an economy as well as market timing. Bali et al. (2009), Breckenfelder and Tedongap (2012) and Sevi (2013), use different risk measures to examine the marginal effect of downside risk on expected stock returns, and find a positive and significant risk-return relation in which a risk-averse investor requires compensation for downside risk but is willing to pay a discount/insurance-fee for upside gain.[6]

Our paper is motivated by the above literature that argues, on both empirical and theoretical grounds, for a relationship between risk and return that is nonlinear and possibly time-varying. However, in contrast with the recent studies mentioned above, we revisit the risk-return trade-off in light of the recent theoretical results in Christoffersen and Diebold (2006), who find a direct connection between asset return volatility dependence and asset return sign dependence.

The results in Christoffersen and Diebold (2006) suggest that volatility dependence produces sign dependence and so one should expect sign dependence given the overwhelming evidence of volatility dependence in asset returns. Using this idea, Anatolyev and Gospodinov (2010) decompose excess return into a product of sign and absolute value components to incorporate important nonlinearities in excess return dynamics. Utilizing this decomposition modeling approach, Anatolyev and Gospodinov (2010) obtain statistically and economically significant gains in forecasting financial returns.

In this paper, based on the decomposition modeling approach of Anatolyev and Gospodinov (2010), we propose a new risk-return relation that is capable of capturing dynamic movements and nonlinearities associated with the interdependence between return volatilities and signs. To capture the possible nonlinear dependence between absolute return and sign, a copula function combined with marginals is considered as their joint distribution. As discussed in Liu (2015), the skewness of financial returns can be characterized naturally by the dependence between absolute return and sign. In this regard, the decomposition model might provide us an alternative method to account for the asymmetry of the risk-return relation recently found in literature.[7]

To fix ideas, we decompose the excess return, r_t, into the product of absolute value and sign

$$r_t = |r_t| sign(r_t) \tag{1.1}$$

which is called "an intriguing decomposition" in Christoffersen and Diebold (2006). From (1.1), the conditional mean of r_t can be expressed in terms of an indicator function, $s_t = \mathbb{I}(r_t > 0)$, as

$$E_{t-1}(r_t) = 2E_{t-1}(|r_t| s_t) - E_{t-1}(|r_t|) \tag{1.2}$$

which consists of the conditional expectation of the product between absolute return and sign, and the conditional expectation of absolute return. The conditional dynamics of absolute return can be specified using the multiplicative error modeling (MEM) framework of Engle (2002) as $|r_t| = \psi_t \eta_t$, with $\psi_t = E_{t-1}(|r_t|)$ and η_t is a positive multiplicative error with $E_{t-1}(\eta_t) = 1$.

Following Ghysels et al. (2005) and Ghysels et al. (2014), we consider absolute returns as the proxy for market risk in our decomposition model (1.1) and (1.2). Setting $V = \psi_t$ in the general definition of the partial risk-return relation and taking the

[2] See e.g., Ghysels et al. (2005), Guo and Whitelaw (2006), Ludvigson and Ng (2007), Kanas (2012), Bollerslev et al. (2013), Kinnunen (2014), among many others.

[3] See e.g., Brandt and Wang (2010), Lettau and Ludvigson (2010), Bollerslev et al. (2011), Ghysels et al. (2014), among many others.

[4] See e.g., Brunnermeier and Nagel (2008), Ghysels et al. (2014), Kinnunen (2014), Salvador et al. (2014), Wu and Lee (2015), among many others.

[5] In addition to the linearity restriction, the use of variance as the proxy for risk in Merton (1973) is a natural result under the assumption of normality. However, this is quite restrictive due to the stylized fact of non-Gaussian financial data.

[6] For more studies in nonlinear risk-return trade-offs, see also Hansen and Scheinkman (2009), Lustig and Verdelhan (2012), Wang and Yang (2013), Bonomo et al. (2015), etc.

[7] In addition to the recent studies discussed earlier, Cheng and Jahan-Parvar (2014) also show the empirical evidence for the necessity of modeling conditional skewness in the risk-return relation by allowing for non-zero conditional skewness in the returns of fourteen Pacific basin equity markets.