Journal of Economic Dynamics & Control 60 (2015) 95–111

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Solving generalized multivariate linear rational expectations models

Fei Tan [a,b], Todd B. Walker [c,*]

[a] Interdisciplinary Center for Social Sciences, Zhejiang University, 38 Zheda Rd, Hangzhou 310027, China

[b] Department of Economics, John Cook School of Business, Saint Louis University, 3674 Lindell Boulevard, St. Louis, MO 63108-3397, United States

[c] Department of Economics, Indiana University, Wylie Hall Rm105, 100 SouthWoodlawn, Bloomington, IN 47405, United States

A R T I C L E I N F O

Article history:
Received 4 March 2014
Received in revised form
16 July 2015
Accepted 27 July 2015
Available online 25 August 2015

JEL classification:
C32
C62
C65
E63

Keywords:
Solution methods
Analytic functions
Rational expectations

A B S T R A C T

We generalize the linear rational expectations solution method of Whiteman (1983) to the multivariate case. This facilitates the use of a generic exogenous driving process that must only satisfy covariance stationarity. Multivariate cross-equation restrictions linking the Wold representation of the exogenous process to the endogenous variables of the rational expectations model are obtained. We argue that this approach offers important insights into rational expectations models. We give two examples in the paper—an asset pricing model with incomplete information and a monetary model with observationally equivalent monetary-fiscal policy interactions. We relate our solution methodology to other popular approaches to solving multivariate linear rational expectations models, and provide user-friendly code that executes our approach.

1. Introduction

Whiteman (1983)lays out a solution principle for solving stationary, linear rational expectations models. The four tenets of the solution principle are: (i) Exogenous driving processes are taken to be zero-mean linearly regular covariance stationary stochastic processes with known Wold representation; (ii) expectations are formed rationally and are computed using Wiener-Kolmogorov formula; (iii) solutions are sought in the space spanned by time-independent square-summable linear combinations of the process fundamental for the driving process; (iv) the rational expectations restrictions are required to hold for all realizations of the driving processes. The purpose of this paper is to extend Whiteman's solution principle to the multivariate setting.

The solution principle is general in the sense that the exogenous driving processes are assumed to only satisfy covariance stationarity. Solving for a rational expectations equilibrium is nontrivial under this assumption and Whiteman demonstrates how powerful z-transform techniques can be used to derive the appropriate fixed point conditions.

* Corresponding author.
E-mail addresses: tanf@indiana.edu (F. Tan), walkertb@indiana.edu (T.B. Walker).

http://dx.doi.org/10.1016/j.jedc.2015.07.007
0165-1889/

The techniques advocated in Whiteman (1983) are not well-known. This could be because the literature contains several well-vetted solution procedures for linearized rational expectations models (e.g., Sims, 2001b; Anderson, 2006) or because the solution procedure requires working knowledge of concepts unfamiliar to economists (e.g., z-transforms). We provide an introduction to these concepts and argue that there remain several advantages of Whiteman's approach on both theoretical and applied grounds. First, the approach only assumes that the exogenous driving processes possess a Wold representation, allowing for a relaxation of the standard assumption that exogenous driving processes follow an autoregressive process of order one, AR(1), specification. As recently emphasized in Curdia and Reis (2012), no justification is typically given for the AR (1) specification with little exploration into alternative stochastic processes despite obvious benefits to such deviations.[1] Second, models with incomplete information or heterogeneous beliefs are easier to solve using the z-transform approach advocated by Whiteman. Kasa (2000) and Walker (2007) show how these methods can be used to generate analytic solutions to problems that were approximated by Townsend (1983) and Singleton (1987).[2] Third, as shown in Kasa (2001) and Lewis and Whiteman (2008), the approach can easily be extended to allow for robustness as advocated by Hansen et al. (2011) or rational inattention as advocated by Sims (2001a). Finally, there are potential insights into the econometrics of rational expectations models. Qu and Tkachenko (2012) demonstrate how working in the frequency-domain can deliver simple identification conditions.

The contribution of the paper is to extend the approach of Whiteman (1983) to the multivariate setting and (re)introduce users of linear rational expectations models to the analytic function solution technique. We provide sufficient (though not exhaustive) background by introducing a few key theorems in Section 2.1 and walking readers through the univariate example of Whiteman, 1983 in Section 2.2. Section 3 establishes the main result of the paper. There is a chapter devoted to multivariate analysis in Whiteman (1983) that has known errors (see Onatski, 2006; Sims, 2007). Section 3.3 provides an example of these errors and demonstrates why our approach does not suffer from the same setback. In effect, our approach is a straightforward way to maintain the methodology of Whiteman by providing robust existence and uniqueness criteria. Finally, Section 4 provides a few examples that demonstrate the usefulness of solving linear rational expectations models in the frequency-domain. An online Appendix B provides a user's guide to the MATLAB and Maple code that executes the solution procedure. To the best of our knowledge, our symbolic code, along with the Anderson–Moore Algorithm (Anderson and Moore, 1985; Anderson, 2006), is the only publicly available code that symbolically solves for rational expectations equilibria. The code is available at http://www.pages.iu.edu/walkertb/.

2. Preliminaries

Elementary results concerning the theory of stationary stochastic processes and the residue calculus are necessary for grasping the z-transform approach advocated here. This section introduces few important theorems that are relatively well-known but is by no means exhaustive. Interested readers are directed to Brown and Churchill (2013) and Whittle (1983) for good references on complex analysis and stochastic processes, and Kailath (1980) for results on matrix polynomials. Sargent (1987) provides a good introduction to these concepts and discusses economic applications.

2.1. A few useful theorems

The first principle of Whiteman's solution procedure assumes that the exogenous driving processes are zero-mean linear covariance stationary stochastic processes with no other restrictions imposed. The Wold representation theorem allows for such a general specification.

Theorem 1 (*Wold Representation Theorem*). *Let $\{x_t\}$ be any $(n \times 1)$ covariance stationary stochastic process with $\mathbb{E}(x_t) = 0$. Then it can be uniquely represented in the form*:

$$x_t = \eta_t + A(L)\varepsilon_t \tag{1}$$

where $A(L)$ is a matrix polynomial in the lag operator with $A(0) = I_n$ and $\sum_{s=1}^{\infty} A_s A_s'$ is convergent. The process ε_t is n-variate white noise with $\mathbb{E}(\varepsilon_t) = 0$, $\mathbb{E}(\varepsilon_t \varepsilon_t') = \Sigma$ and $\mathbb{E}(\varepsilon_t \varepsilon_{t-m}') = 0$ for $m \neq 0$. The process ε_t is the innovation in predicting x_t linearly from its own past

$$\varepsilon_t = x_t - \mathbb{P}[x_t | x_{t-1}, x_{t-2}, \ldots] \tag{2}$$

where $\mathbb{P}[\cdot]$ denotes linear projection. The process η_t is linearly deterministic; there exists an n vector c_0 and $n \times n$ matrices C_s such that without error $\eta_t = c_0 + \sum_{s=1}^{\infty} C_s \eta_{t-s}$ and $\mathbb{E}[\varepsilon_t \eta_{t-m}'] = 0$ for all m.

The Wold representation theorem states that *any* covariance stationary process can be written as a linear combination of a (possibly infinite) moving average representation where the innovations are the linear forecast errors for x_t and a process

[1] This is true despite the fact that Kydland and Prescott (1982), the paper that arguably started the real business cycle literature, contains an interesting deviation from the AR(1) specification.

[2] Taub (1989), Kasa et al. (2014), Rondina (2009), and Rondina and Walker (2013) also use the space of analytic functions to characterize equilibrium in models with informational frictions. Seiler and Taub (2008), Bernhardt and Taub (2008), and Bernhardt et al. (2010) show how these methods can be used to accurately approximate asymmetric information equilibria in models with richer specifications of information.