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Using nonlinear model predictive control for dynamic decision problems in economics

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ABSTRACT

This paper presents a new approach to solve dynamic decision models in economics. The proposed procedure, called Nonlinear Model Predictive Control (NMPC), relies on the iterative solution of optimal control problems on finite time horizons and is well established in engineering applications for stabilization and tracking problems. Only quite recently, extensions to more general optimal control problems including those appearing in economic applications have been investigated. Like Dynamic Programming (DP), NMPC does not rely on linearization techniques but uses the full nonlinear model and in this sense provides a global solution to the problem. However, unlike DP, NMPC only computes one optimal trajectory at a time, thus avoids to grid the state space and for this reason the computational demand grows much more moderately with the space dimension than for DP. In this paper we explain the basic idea of NMPC, give a proof concerning the accuracy of NMPC for discounted optimal control problems, present implementational details, and demonstrate the ability of NMPC to solve dynamic decision problems in economics by solving low and high dimensional examples, including models with multiple equilibria, tracking and stochastic problems.

1. Introduction

The lack of closed form solutions of dynamic decision models with optimizing agents has generated a large number of computational methods to solve such models. A detailed discussion of a variety of numerical methods and accuracy tests are provided in Santos and Vigo-Aguiar (1998), Judd (1998), Juillard and Villemot (2011) and Grüne and Semmler (2004). The latter have proposed Dynamic Programming (DP), with grid refinement, cf. Grüne (1997), to solve a family of continuous and discrete time dynamic models with optimizing agents. DP provides the value function and the control variable in feedback form, even for rather complex problems.

In DP a global solution to the optimal control problem is found by first computing an approximation to the optimal value V and then computing the optimal control from V, see Grüne and Semmler (2004). Yet, since DP computes the value and policy function at each point of a grid of the state space, it has the disadvantage that even with an adaptive choice of the grid its numerical effort typically grows exponentially with the dimension of the state variable. This disadvantage is independent

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of the actual solution technique for the DP equation, be it value iteration, policy iteration or other any other method. Hence, already for moderate state dimensions it may be impossible to compute a solution with reasonable accuracy.

This paper illustrates how Nonlinear Model Predictive Control (NMPC) can be used as an alternative approach to solve dynamic decision models in economics. NMPC is a well-known method in control engineering which is frequently used in industrial practice, particularly in chemical process engineering. Traditionally, NMPC is applied to optimal feedback stabilization problems, see, e.g., Rawlings and Mayne (2009) or Grüne and Pannek (2011) and the references therein. Recently, however, the application of NMPC to more general optimal control problems has attracted considerable attention, see, e.g., Amrit et al. (2011), Angeli et al. (2009), Angeli and Rawlings (2010), Diehl et al. (2011), Grüne (2013), and Grüne and Stieler (2014) for undiscounted optimal control problems. Similar to DP, NMPC can solve nonlinear dynamic decision problems globally without having to resort to local approximations by linearization techniques.

However, unlike DP the solution is not found on a grid in state space. Rather, an infinite horizon trajectory is synthesized by putting together pieces of finite horizon optimal trajectories, which implies that the numerical effort of the approach scales much more moderately with the state dimension. This approach, termed *receding horizon control* in control engineering, is in fact not unknown in economics. In the economic literature, it is known as *sliding or rolling planning*, see, e.g., Kaganovich (1985) and the references therein. However, in the economic context we are only aware of applications of this approach to linear models. The contribution of this paper is to demonstrate that NMPC also applies to nonlinear problems in dynamic decision making in economics. To this end, we establish a convergence result for discounted optimal control problems and illustrate its performance by applying it to several economic decision models.

Assuming that a reliable numerical solver for finite horizon optimal control problems is available,[1] the main source of errors in NMPC is the difference between the optimal trajectories of finite and infinite horizon optimal control problems. In Section 3, we show that for discounted optimal control problems with small discount factor and for problems satisfying the so-called turnpike property, cf. McKenzie (1986), for sufficiently long finite horizons, NMPC yields approximately infinite horizon optimal trajectories. Unlike other numerical errors, like, e.g., interpolation errors in DP, this source of errors allows for a precise economic interpretation. In fact, the mismatch between the true solution of an infinite horizon decision problem and its NMPC solution is due to the fact that the decision for the control to be implemented at the next time step is taken by looking at the problem on a truncated time horizon, i.e., with a particular form of decision making under limited information.

Sims (2005, 2006), in a series of research papers, showed that agents make decisions under limited information: the information is either not available or the agents respond imprecisely to the available information. In this context, we can interpret the gap between the infinite horizon solution and the NMPC solution as induced by the agents' decision making using only limited information. As such, the abstract convergence results from Section 3 have a self-evident economic interpretation: if the agents' information and information processing capacity increase then it is likely to better approximate the infinite horizon decision making,[2] which is reflected, e.g., in the examples in Sections 4.1 and 5.1 below. One might even use NMPC to systematically study the effects of decision making for this particular form of rational inattention.

Though we do not elaborate further on the latter aspect in this paper, we would like to make some remarks on the finiteness of the decision horizon. The argument could be made that if the agents come close to the final period they will sell all their assets which will impact the last period's outcome. Yet, the way the NMPC solution procedure is set up, only the first decision step is implemented. If the decision horizon is N, then one is, in the closed loop solution, always $N-1$ periods away from the final decision. Hence, one never sees the effects which appear at the end of the decision horizon. This can be formalized using the turnpike property, which then allows us to prove a formal convergence result, see Section 3. Under the appropriate conditions, this property holds without using the salvage value of the finite horizon model. If, however, the decision horizon is short it might be beneficial to take into account the salvage value, provided it can be determined in a reasonable way. Likewise, information about optimal steady states may be incorporated into the NMPC algorithm via terminal constraints which may be useful for short decision horizons. Yet, as the decision horizon becomes larger, typically there is no need for taking the salvage value or information about steady state into account since NMPC already approximates well the infinite horizon decision model. One of the issues will thus be how large the decision horizon N needs to be, see Sections 3 as well as the discussion for the examples in Sections 4.1 and 5.1.

In this paper, we evaluate the performance of NMPC analytically and via computer simulations for a selection of dynamic decision models in economics. Particularly, we extend the economic MPC results from the literature by considering discounted optimal control problems, both analytically and by studying a number of examples by means of numerical simulations. In order to study the accuracy of NMPC for approximating discounted infinite horizon problems, we first want to test our algorithm by studying the well-known basic growth model of Brock and Mirman (1972) type, for which the exact solution is known, and a recent DSGE extension of it. To study the Brock et al. model allows us to judge the accuracy of our numerical method for a model with short decision horizon, and to explore what the new method can contribute.

As mentioned above, there are, in the economic literature, more complicated dynamic models with optimizing agents which have been a challenge to commonly used numerical techniques. These are models with multiple equilibria, regime

[1] For a discussion of this aspect see Section 7.

[2] Sims notes "… the capacity-constrained agent's behavior approximates that of a fully optimizing agent, but with a tight capacity constraint his behavior will be much more weakly correlated with external information than the behavior of a fully optimizing agent would be (Sims, 2006, p. 158)".