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Discrete-time behavioral portfolio selection under cumulative prospect theory ☆

Yun Shi [a], Xiangyu Cui [b], Duan Li [c,*]

[a] School of Management, Shanghai University, China
[b] School of Statistics and Management, Shanghai Key Laboratory of Financial Information Technology, Shanghai University of Finance and Economics, China
[c] Department of Systems Engineering & Engineering Management, The Chinese University of Hong Kong, Hong Kong

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ABSTRACT

We formulate and study three multi-period behavioral portfolio selection models under cumulative prospect theory: (i) S-shaped utility maximization without probability weighting in a market with one risky asset; (ii) S-shaped utility maximization without probability weighting in a market with multiple risky assets which follow a joint elliptical distribution; and (iii) S-shaped utility maximization with inverse-S-shaped probability weighting in a market with one risky asset. For the first two time consistent models, we identify the well-posedness conditions and derive the semi-analytical optimal policies. For the third time inconsistent model, we assume that the investor is aware of the time inconsistency but is unable to commit to his initial plan of action. Then, we reformulate the model into an intrapersonal game model and derive the semi-analytical subgame perfect Nash equilibrium (time consistent) policy under well-posedness condition. All the three policies take a piece-wise linear feedback form. Our analysis of the three models not only partially explains the well documented phenomena of non-participation puzzle and horizon effect, but also extends the two fund separation theorem into multi-period S-shaped utility setting and pushes forward the study on time inconsistency issue incurred by probability weighting.

1. Introduction

Neoclassical finance theory, which has been occupying a dominating position in interpreting phenomena in the financial world since its birth in the 1950s, views the market participants as *rational* "wealth maximizers". On the other hand, behavioral finance, which has been developing since the 1980s, posits that emotion and psychology influence investors and thus create market anomalies labeled as *irrational* (bounded rational) behaviors. However, to this date, the theoretical research in the quantitative finance (financial engineering) community has been still largely confined to the realm of neoclassical finance.

This paper aims at building up a series of behavioral portfolio models in discrete-time setting which capture prominent behavioral features of investors revealed in behavioral finance and incorporate the market incompleteness in general

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* Corresponding author.
E-mail address: dli@se.cuhk.edu.hk (D. Li).

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situations. Prospect theory (PT, or CPT) (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), the representative behavioral theory, employs cognitive psychological features to incorporate irrational (bounded rational) human behavior into economic decision making. Its four key elements in the context of financial asset allocation are that (i) people evaluate assets on gains and losses (which are defined with respect to a reference point), (ii) the degree of suffering due to loss is more than the degree of happiness due to gain with a similar magnitude (loss aversion), (iii) people are risk-averse for gains with moderate to high probabilities and losses with small probabilities while risk-seeking for losses with moderate to high probabilities and gains with small probabilities, and (iv) people do not weight outcomes by their objective probabilities, but rather by transformed probabilities or decision weights. In technical terms, these elements together give rise to S-shaped utility function (partly convex and partly concave) and inverse-S-shaped probability weighting function in the preference measure, which cause the behavioral portfolio model nonconvex and time inconsistent. These new features, in turn, render an inevitable failure of the convex duality and dynamic programming, the two main, if not the only, approaches in solving dynamic expected utility models in neoclassical finance.

A dynamic behavioral portfolio model, unlike the classical utility maximization model, could be easily ill-posed and time inconsistent. Roughly speaking, the ill-posedness issue arises when the utility associated with gains substantially outweighs the dis-utility associated with losses, especially for large payoffs. Based on these observations, one straightforward cure for the ill-posedness issue in the literature is making loss-aversion coefficient large enough to balance the utility associated with large gains and the dis-utility associated with large losses. Based on such an idea, we introduce an *induced loss-aversion* measure for each time period and then give an easily-checking critical threshold for each time period. For a given time period, when the induced loss-aversion measure exceeds the critical threshold level, the problem is well-posed; otherwise, it becomes ill-posed. One of the significant findings of this paper is the monotonicity of the induced loss-aversion measures: the earlier the time period, the lower the induced loss-aversion level for that time period. This recognition explains partially the horizon effect of investors.

On the other hand, probability weighting in a multi-period setting generates a time inconsistency issue. Roughly speaking, probability weighting makes the investor's preference changing in a way that the investor's long-term global preference is not consistent with his short-term local preference, i.e., the preference does not satisfy Bellman's principle of optimality in dynamic programming. In other words, the investor may deviate at the later stages from the global optimal policy which he previously planned to act at time 0. To solve this inconsistency caused by probability weighting, we follow a general approach in handling other time inconsistent dynamic decision problems in the literature (see O'Donoghue and Rabin, 1999; Basak and Chabakauri, 2010; Barberis, 2012; Björk et al., 2014). Under their framework, the investor is assumed to be *sophisticated*: "he is aware of the time inconsistency but is unable to find a way of committing to his initial plan" (Barberis, 2012, p. 37). Then, we can reformulate the behavioral portfolio model as an intrapersonal game, in which the sophisticated investor at any time instance acts as a Stackelberg leader and chooses his "best" policy by taking into account his policies in future periods. The subgame perfect Nash equilibrium policy which the sophisticated investor adopts is termed as *time consistent policy*.

In this paper, we study three behavioral portfolio models: (i) S-shaped utility maximization without probability weighting in a market with one risky asset; (ii) S-shaped utility maximization without probability weighting in a market with multiple risky assets which follow a joint elliptical distribution; and (iii) S-shaped utility maximization with inverse-S-shaped probability weighting in a market with one risky asset. The first two models are time consistent, while the third one is not. Under the well-posedness conditions, we derive the semi-analytical optimal policies for the first two models and the semi-analytical time consistent policy for the third model, which all take a piecewise linear form. Furthermore, in the analysis of model (i), we show that our result partially explains the horizon effect, a long standing puzzle. When the reference point in our model is taken as the wealth level from investing solely in the risk-free asset, our result also sheds light on the well documented anomaly, non-participation effect. In the analysis of model (ii), we demonstrate that the two fund separation is still valid under the S-shaped utility in discrete-time setting. In the analysis of model (iii), we take the first attempt in resolving the time inconsistency issue induced by probability weighting in discrete-time portfolio selection problem. We also show some mixed effects of probability weighting on the horizon effect. When the probability weighting is not severe, the horizon effect exists. While the probability weighting is severe, the horizon effect disappears due to the overweighting of the worst realization caused by the probability weighting.

There has been growing research interest in incorporating (cumulative) prospect theory into a standard portfolio selection model in a discrete-time setting; see, for example, Barberis and Huang (2008), Bernard and Ghossoub (2010), He and Zhou (2011), Pirvu and Schulze (2012), Del Vigna (2013), Barberis and Huang (2009), Barberis and Xiong (2009) and De Giorgi and Legg (2012). However, except for the last three, most of these works have focused on a static or *single-period* setting. In this paper, we consider *multi-period* portfolio choice problems, which allow wealth reallocation during the investment process, thus offering a more realistic policy than the static one (buy-and-hold policy). As we mentioned above, the works in He and Zhou (2011), Pirvu and Schulze (2012), and Barberis and Xiong (2009) are most related to our models.

He and Zhou (2011) studied single-period S-shaped utility maximization with probability weighting in a market with one risky asset. Model (iii) in this paper represents a multi-period counterpart of the model in He and Zhou (2011). However, due to the time inconsistency of model (iii), we adopt an intrapersonal game formulation in deriving the semi-analytical sub-game Nash equilibrium (time consistent) policy. This is the first attempt in the literature to deal with time inconsistency issue induced by probability weighting in discrete-time portfolio selection problem. Pirvu and Schulze (2012) considered single-period S-shaped utility maximization with probability weighting in a market with multiple risky assets which follow a joint elliptical distribution, and proposed a two-fund separation between the riskless asset and the market