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On increasing risk, inequality and poverty measures: Peacocks, lyrebirds and exotic options

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ABSTRACT

We extend the Rothschild and Stiglitz (1970) notion of increasing risk to families of random variables and in this way link their approach to the concept of stochastic processes which are increasing in the convex order. These processes have been introduced in seminal work by Strassen (1965), Doob (1968) and Kellerer (1972), who showed that such processes have the same marginals as a martingale. In fact, we demonstrate that their results include the results of Rothschild and Stiglitz as a special case. Further, we show that it makes sense to look at a larger class of processes, which we refer to as lyrebirds. We also show how these processes link up with the concept of second order stochastic dominance and are helpful in studying the dynamics of inequality and poverty measures. Further applications discussed include geometric and hyperbolic discounting, exotic derivatives and real options.

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1. Introduction

It is without doubt that random variables and their associated distributions play a fundamental role in economic modeling. For decision making and policy guidance it is tremendously important to be able to classify the degree of variability of random variables and in particular what it means that one random variable, say Y, is more variable than another random variable, say X. The realization of this variability can be multitude and be manifested in say the returns of a financial asset, in which case we would link variability to risk, or the income or wealth of an arbitrarily chosen individual among a certain population, in which case we would link variability to inequality. Rothschild and Stiglitz (1970, 1971) contributed in a fundamental way to the characterization of variability in the context of risk. They formalized in a mathematical rigorous way what it means for one random variable to be more risky than another and

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showed the following: Let X and Y be two sufficiently integrable random variables,[2] then the two statements below are equivalent:

$\mathcal{A}1$: $Y \overset{law}{=} X+Z$ (equal in law) where Z is a random variable with the property that

$$\mathbb{E}(Z|X)=0 \tag{1}$$

$\mathcal{A}2$: For all concave (utility) functions $U(\cdot)$ s.t. $\mathbb{E}(U(X))$ and $\mathbb{E}(U(Y))$ are both finite

$$\mathbb{E}(U(X)) \geq \mathbb{E}(U(Y)). \tag{2}$$

Intuitively, the first statement says that the distribution of the random variable Y is like the distribution of X with additional risk which is unaffected by X; hence Y is riskier than X. The intuition of the second statement is that every risk averse decision maker prefers X to Y.

In the context of inequality measures (as well as in the context of risk) the concept of first and second order stochastic dominance has always played a major role, compare (Levy, 1992). Denoting with ρ_X and ρ_Y the cumulative distribution functions of X and Y, then X dominates Y by second order stochastic dominance, if and only if

$$\int_{-\infty}^{x} \rho_X(t)\,dt \leq \int_{-\infty}^{x} \rho_Y(t)\,dt \tag{3}$$

for all $x \in \mathbb{R}$, provided that the two integrals exist and are finite. A straightforward integration by parts gives

$$\int_{-\infty}^{x} \rho_X(t)\,dt = \int_{-\infty}^{x} (x-t)\,d\rho_X(t) = \mathbb{E}\big((x-X)^+\big), \tag{4}$$

where $(x-X)^+ = \max(x-X,0)$. If we assume that $E(X)=E(Y)$, then $(x-X)^+ - x = (X-x))^+ - X$ implies that second order stochastic dominance of X over Y is equivalent to

$\mathcal{A}3$: For all $x \in \mathbb{R}$ we have

$$\mathbb{E}\big((X-x))^+\big) \leq \mathbb{E}\big((Y-x)^+\big). \tag{5}$$

In fact under the condition $\mathbb{E}(X)=\mathbb{E}(Y)$ it can be shown that the statement $\mathcal{A}3$ is equivalent to $\mathcal{A}2$. Without imposing equality in expectations of X and Y condition $\mathcal{A}3$ becomes equivalent to the relaxation of condition $\mathcal{A}2$, when it is only imposed upon increasing concave functions, a condition which we will later refer to as $\mathcal{D}2$. This can be found for example in Atkinson (1970). Further, if the random variables X and Y above are positive, and condition (5) in $\mathcal{A}3$ is valid for all $x \geq 0$, then it is automatically satisfied for all $x \in \mathbb{R}$, a fact that we will use later when we will be looking at various inequality measures.

The linkage between Rothschild and Stiglitz's (1970, 1971) work and the work following Atkinson (1970) with the theory of derivatives and option pricing becomes evident in Eq. (5), where the expressions on both side represent the prices of European call options written on the underlying X resp. Y with strike price x. Since its inception in the early 1970s[3] option pricing theory has developed sophisticated methods to evaluate expressions such as those in Eq. (5) and in particular study their dependence on certain parameters within the model, which is related to the determination of the so-called Greeks of an option. It is therefore surprising that this approach so far has not been adopted more consistently in the literature.

In this paper we demonstrate how methodology that has classically appeared in the context of option pricing as well as more or less abstract probability theory can be helpful in the context of Rothschild and Stiglitz (1970, 1971) approach to classify risk as well as within the large literature on inequality and poverty, dating back to Atkinson (1970, 1987). More specifically, we extend the Rothschild and Stiglitz (1970, 1971) notion of increasing risk to families of random variables (X_t) and in this way link their approach to the concept of stochastic processes which are increasing in the convex order. These processes, now referred to as peacocks, have recently received increased attention in the derivatives pricing literature, compare (Hirsch et al., 2011). Originally these processes were introduced in seminal work by Strassen (1965); Doob (1968) and Kellerer (1972), who showed that such processes have the same marginals as a martingale. As we show, their results in fact include the results of Rothschild and Stiglitz as a special case. Further, we demonstrate that it makes sense to look at a larger class of processes, which we refer to as lyrebirds and which nicely link to Atkinson (1970) work. We show how these processes link up with the concept of second order stochastic dominance, which opens up the gate for exciting applications

[2] Rothschild and Stiglitz (1970, 1971) only consider random variables which map into the compact interval $[0,1]$. In this way they circumvent many problems that occur in the general case. A quote from their paper says "The extension (and modification) of the results to c.d.f.s defined on the whole real line is an open question whose resolution requires the solution of a host of delicate convergence problems of little economic interest." Contrary to their statement however, most applications in Economics and Finance involve random variables with arbitrary and unbounded domains. Fortunately, the delicate convergence problems have been solved by Strassen (1965), Doob (1968) and Kellerer (1972) to which we revert in the next section.

[3] Interestingly this is at about the same time when Rothschild and Stiglitz as well as Atkinson started their work.

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