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Learnability of an equilibrium with private information

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ABSTRACT

This paper investigates the learnability of an equilibrium with private information. Agents of each type have their own private information about an exogenous variable and conduct adaptive learning with a heterogeneously misspecified perceived laws of motion (PLM) that includes only this variable. The paper shows that the existence of private information has a nonnegative impact on the learnability of the equilibrium; that is, the condition for learnability is unaffected or relaxed by heterogeneity and/or misspecification in PLMs caused by private information. In a New Keynesian model with private information about fundamental shocks, the learnability of the equilibrium is ensured by the Taylor principle of monetary policy. The paper also confirms that these results hold true not only in the presence of private information, but also in a variety of informational structures.

1. Introduction

Rational expectations are based on an unlikely assumption that agents have perfect knowledge about the structure of the economy. Recent research has incorporated the concept of *adaptive learning* as an alternative framework in which agents formulate their forecasts by estimating econometric models (that is, perceived laws of motion, PLMs) (see Bray, 1982; Evans and Honkapohja, 2001). The issue to be addressed in this framework is whether agents' forecasts converge around an equilibrium; that is, whether the equilibrium is *learnable*, and a number of papers have investigated macroeconomic policies ensuring the learnability of the equilibrium (e.g., Bullard and Mitra, 2002; Evans and Honkapohja , 2003a,b).

Learnability has been analyzed in different structures of information sets of economic variables held by agents. Evans and Honkapohja (2001) provide a benchmark framework where each agent has the full information set of economic variables to form a correctly specified PLM including all relevant variables (hereafter, CS learning). Branch (2004) and Guse (2008) focus on the *restricted perceptions equilibrium* (hereafter, RPE) where agents' information sets are limited so that they are constrained to form underparameterized PLMs excluding unobservable variables. Adam et al. (2006) and Berardi (2007) introduce heterogeneity in agents' information sets, where a fraction of agents have limited information sets while other agents have full information sets.

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This paper investigates the learnability of an equilibrium in the presence of private information. Agents have private information about exogenous variables, which makes agents' information sets limited and heterogeneous.[1] When agents make forecasts, each type of agent is constrained to form a *heterogeneously misspecified* PLM that includes only her own observable variable (hereafter, HM learning). In this economy, there emerges a generalization of the RPE, which we call a *heterogeneous misspecification equilibrium* (hereafter, HME). The paper clarifies the learnability of the HME and investigates whether and how the learnability is affected by the existence of private information. Next, in a basic New Keynesian (NK) macroeconomic model with private information about fundamental shocks, this paper provides learnability conditions imposed on monetary policy with a Taylor-type interest rate rule.

The result of the paper is that the existence of private information has a nonnegative impact on the learnability of an equilibrium; that is, the condition for learnability is unaffected or relaxed by the existence of private information. Specifically, this impact is nondecreasing in the degrees of heterogeneity and misspecification in PLMs caused by private information. The paper also confirms that these results hold true not only in the presence of private information, but also in a variety of informational structures considered in the literature. Next, in the NK model, the learnability condition under HM learning imposed on monetary policy is not more restrictive than the condition under CS learning, that is, the *Taylor principle*—to raise nominal interest rates more than one-for-one in response to an increase in inflation (Taylor, 1993). Calibrations show that the impact of private information is economically significant in the NK model, and that if lagged endogenous variables are included in PLMs as public information, the impact is reduced, but remains significant.

These results are intuitive as limited and/or heterogeneous information about fundamentals have been widely recognized to cause the reduced and slow adjustment of the economy (e.g., Calvo, 1983; Veldkamp, 2005), which makes it easy to learn the dynamics of the economy. This mechanism is consistent with the learning literature that explains the observed persistence of the economy using heterogeneity and/or misspecification in PLMs (e.g., Adam, 2007; Slobodyan and Wouters, 2012; Hommes and Zhu, 2014). Our results reinforce the Bullard and Mitra (2002)'s finding (that is, the Taylor principle is a sufficient condition for the learnability of an equilibrium) to be robust to the existence of private information, and they also complement the Honkapohja and Mitra (2006)'s finding that heterogeneity does not make learnability conditions more restrictive.

This paper is closely related to the learning literature regarding the presence of private information. Marcet and Sargent (1989a) establish learning schemes for an equilibrium with private information. Branch and McGough (2011) study business cycle dynamics under adaptive learning with private information. Heinemann (2009) considers the existence of private noisy signals of economic variables. It has not, however, been investigated whether/how the private information that continuously makes agents' PLMs heterogeneously misspecified has an impact on learnability.

Our analysis is also related to the literature on heterogeneity and misspecification in learning. Adam et al. (2006) and Berardi (2007, 2009) consider heterogeneous and misspecified learning where a fraction of agents form underparameterized PLMs, while other agents form correctly specified PLMs. Their models may be interpreted as allowing the partial existence of private information, but they do not include a plausible case where each type of agent has her own private information.[2] Meanwhile, the heterogeneously misspecified PLMs in this paper are also considered in the context of the literature introducing *dynamic predictor selection*, which was originally established by Brock and Hommes (1997). Branch and Evans (2006, 2007), for example, allow agents to choose among a list of heterogeneously misspecified PLMs. However, their model premises homogeneity in agents' information sets so that all agents have the same list of PLMs, and this homogeneity conflicts with the existence of private information that causes intrinsic heterogeneity in information sets and constrains each agent to hold a specific heterogeneously misspecified PLM. Thus, the HM learning analyzed in this paper describes adaptive learning with private information more suitably.

The paper is structured as follows. The next section presents our model and provides a benchmark analysis about learning without private information. Section 3 introduces learning with private information and investigates the dynamics of the HME. Section 4 examines the impact of private information on the learnability of an equilibrium. Section 5 evaluates numerically the impact in a basic NK model. Finally, we present our conclusions.

2. Model

2.1. Setup

We establish the general form of the multivariate linear expectations model. The economy is represented by two vector equations:

$$y_t = A + BE_t^* y_{t+1} + Cw_t, \tag{1}$$

$$w_t = \Phi w_{t-1} + v_t. \tag{2}$$

[1] For example, a preference shock possessed by a household continues to be observable only for this household (see Allen and Gale, 2004). In financial markets, the profitability of a borrower tends to be observable only by this borrower (see Stiglitz and Weiss, 1981). Branch (2007) provides evidence that information sets are limited and heterogeneous using the Michigan survey of inflation expectations. Bovi (2013) uses the survey data of the European Commission to show that heterogeneous beliefs are persistent in UK citizens.

[2] Honkapohja and Mitra (2004b) and Muto (2011) consider that the private sector and the central bank specify mutually different PLMs. However, their models include not only the heterogeneity in PLMs but also structural heterogeneity, and hence the effect of the former heterogeneity is not identified.