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Convergence of optimal harvesting policies to a normal forest

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ABSTRACT

This paper extends the forestry maximum principle of Heaps (1984) to allow the benefits of harvesting to be the utility of the volume of the wood harvested as in Mitra and Wan (1985, 1986). Unlike those authors, however, time is treated as a continuous rather than as a discrete variable. Existence of an optimal harvesting policy is established. Then necessary conditions are derived for the extended model which are also sufficient. The conditions are used to show that under certain boundedness conditions, sequences of optimal harvesting policies contain subsequences which converge pointwise a.e. and in net present value to an optimal harvesting policy. This result is then used to show that any optimal logging policy must converge in harvesting age to a constant rotation period given by modified Faustmann formula. The associated age class distribution converges to a normal forest.

1. Introduction

The optimal harvesting of a multiple age class forest system has received much attention in the forestry economics literature.[1] Here the case of a single species forest of fixed area is considered where the land covered by the forest is homogeneous with respect to its biological and economic characteristics. An important consideration has been to try to determine the asymptotic properties of the age distribution of the forest if it is harvested in a way to maximize the PV of the utility of timber volumes net of logging costs.[2] This paper presents a model where the optimal age distribution converges in the long run to a normal forest with the number of age classes given by a modified Faustmann formula.[3]

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[1] The focus here will be on the economic value of the timber harvested from a forest. However, harvest polices must also in practice take into account the nonmarketed, ecological, social and cultural services provided by forests to society and the value of nontimber products. A brief review of the historical context in which the debate on harvest rates has taken place is in Task Force on Crown Timber Disposal (1974). Kant and Alavalapati (2014) contains many chapters reflecting on noneconomic concerns in sustainable forest management. Rosser (2013) illustrates the complexity of the management problem when some of these concerns are taken into account. The economics literature is reviewed by Tahvonen (2004) who also has a section on the trade off between timber production and protecting natural forest environments.

[2] Abbreviations used in this paper are PV for present value, EXFMP for extended forestry maximum principle and LDCT for Lebesgue dominated convergence theorem.

[3] The age distribution of a normal forest is a uniform distribution. That is it has the same area covered by trees of each age a where $0 \le a \le a_o$ where a_o is the age of the oldest trees in the forest. If at each time the land covered by trees of age a_o is harvested and reforested, then this type of forest will be sustained. Sustainability has been a guiding principle of forest management since at least the 18th century (Amacher et al., 2009, 4).

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The model discussed here is an extension of the forestry maximum principle of Heaps (1984) where time is treated as a continuous variable and optimal logging policies are described by a maximum principle involving delay differential equations. Other authors have set up this model using discrete time and zero harvesting costs.[4] It has not, however, been possible to address the convergence question in a fully satisfactory manner with this type of model (Amacher et al., 2009, 214). Attempts to do so have included making the land area variable (Sahashi, 2002) or allowing for multiple species (Piazza, 2009). The model has also been set up as a distributed control problem in continuous time where the dynamics of age distribution change are set up as a partial differential or integrodifferential equation. There are maximum principles that determine necessary conditions for finding the optimal solution to such problems (Veliov, 2008; Hritonenko and Yatsenko, 2010). These approaches have the advantage of allowing for more realistic modelling of the tree growth process, timber revenues and harvesting costs in terms of their dependence on the age/size characteristics of the trees (Quinn, 1992; Xabadia and Goetz, 2010). There has also been a recent development of numerical methods for solving these types of distributed optimal control problems (Goetz et al., 2011). However, analytical methods for determining the convergence properties of the optimal solutions have yet to be developed.

This paper develops an extended forestry maximum principle (EXFMP) which applies to the maximization of the PV of the utility associated with the harvest minus area dependent harvesting costs. In this framework it is shown that an optimal harvesting policy must involve always harvesting the oldest trees first. Attention can then be restricted to nonnegative real valued functions h with domain $\mathbb{R}_+$ which have the property that the associated discounted function is Lebesgue integrable, i.e. is a member of $L^1_+(e^{-rt})$. The first part of h will represent the initial age distribution of the forest and the second part will represent the distribution over time of the number of hectares logged (and immediately reforested) by a logging policy. It is then shown that the PV function is bounded on the set of harvesting policies. It is further shown that optimal policies belong to a subset of $L^1_+(e^{-rt})$ of policies satisfying certain boundedness conditions. As a consequence, this subset is relatively compact in the weak topology on $L^1_+(e^{-rt})$ and the existence of an optimal logging policy is established as the weak limit of an appropriately chosen sequence of policies in it.

Next the necessary conditions derived in Heaps (1984) are generalized to apply to the extended model. These conditions take the form of delay-differential equations involving both lags and leads. Similar results have been obtained for the related optimal growth models involving vintage capital such as the AK model in Boucekkine and Licandro (2005). Like that model, the conditions obtained are sufficient as well as necessary in characterizing optimal policies.

Finally the EXFMP will be used to show that optimal logging polices must always converge to a constant harvesting policy no matter what the initial age distribution of the forest system is. The key result is to show that if a sequence of optimal logging policies converges to a policy in the weak topology and provided certain boundedness conditions are met, then the limit policy is also optimal and the convergence applies pointwise a.e. and in PV. As time passes, a logging policy changes the age distribution of the trees on the forest land. For an optimal logging policy, the above result is used to show that if the amount of time passed is greater than the length of one complete harvesting cycle, then the new age distribution will be associated with a higher present value of net benefits from future logging than the old age distribution provided the two distributions are different. It was shown in the existence section of the paper that there is one initial age distribution which has a higher present value associated with it than any other age distribution. The corresponding optimal logging policy cannot increase the present value as time passes. This implies that as time passes the age distribution is not changed by this particular logging policy which is only possible if the forest is a normal forest and the policy harvests a constant amount of land as time passes. Further, given any other optimal logging policy, as time passes the PV calculated from the changing age distribution converges to a constant. A final application of the EXFMP is then to show that there is a limiting age distribution which is a normal forest with the harvesting age given by a modified Faustmann formula.[5]

The paper is organized as follows. Section 2 shows that optimal logging policies log first the oldest trees standing in the forest and then sets up the extended model. Section 3 shows that an optimal logging policy exists provided the initial age distribution of the forest is Lebesgue integrable. It also shows that there is one initial age distribution and logging policy that has a higher PV than any other such distribution and policy. Section 4 derives the necessary conditions of the EXFMP and Section 5 then uses the EXFMP to show that optimal logging policies satisfy certain boundedness conditions. It also provides conditions under which weak convergence of a sequence of optimal logging policies implies convergence pointwise a.e. of the associated costate variables. Section 6 then derives the results on the asymptotic convergence of age distributions. Finally, Section 7 concludes the paper.

2. The extended forestry age class model in continuous time

It is desired to devise an optimal logging policy for a forest of total area A which is homogeneous with respect to its biological and economic characteristics. Initially, the area is covered by an uneven aged forest which has $h_o(a)$ hectares of a year old trees in each of the age classes $0 \le a \le a_o$ where $\int_0^{a_o} h_o(a)\, da = A$. Then as time passes the age distribution of the forest changes either by trees becoming older or by land being logged and immediately reforested so that it is now is

[4] For example, Mitra and Wan (1985, 1986), Salo and Tahvonen (2002, 2003), and Khan and Piazza (2012).

[5] Thus the only steady states for the continuous time model are normal forests. In the discrete time model, there may also be steady states involving cycles. In both cases, there may be more than one steady state.