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Pricing external barrier options in a regime-switching model

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ABSTRACT

External barrier options are two-asset options where the payoff is defined on one asset and the barrier is defined on another asset. In this paper, we derive the Laplace transforms of the prices and deltas for the external single and double barrier options where the underlying asset prices follow a regime-switching model with finite regimes. The derivation is made possible because we can obtain the joint Laplace transform of the first passage time of one asset value and the value of the other asset. Numerical inversion of the Laplace transforms is used to calculate the prices of external barrier options.

1. Introduction

Barrier options are path-dependent options that are popular and widely used, because of their flexibility and low premiums. The barrier options become nullified (knock-out or out-options) or activated (knock-in or in-options) when the value of the barrier variable hits a level either from above (down-options) or below (up-options). Merton (1973) derived the analytical valuation of the down-and-out call option. Rubinstein and Reiner (1991) obtained the closed-form pricing formulas for various types of single barrier options. Kunitomo and Ikeda (1992) priced the double barrier options with two curved (exponential) barriers. There have been a large amount of research on pricing barrier options, under various models other than Black–Scholes model. Griebsch (2008) and Chiarella et al. (2012) studied the evaluation of barrier option prices under the Heston stochastic volatility model. Barrier options under the regime-switching model were studied by Henriksen (2011) and Elliott et al. (2014). Barrier options under a discrete time high-order regime-switching model were studied by Ching et al. (2007). For pricing barrier options in the jump diffusion model, refer to the survey paper of Kou (2007). There are also many other papers which study barrier options.

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An external barrier option has a regular call or put payoff from one asset while the barrier crossing is determined by another asset. That is, in an external barrier option, one of the underlying assets determines the actual option payoff and the other asset determines whether the option is knocked in or out. An external barrier option is also known as outside barrier option (Heynen and Kat, 1994), two-asset barrier option (Haug, 1997) or rainbow barrier option (Carr, 1995; Chen et al., 2012). As with standard single barrier options, eight different types of external single barrier options can be identified, depending on the basic type of option (put or call), the nature of the barrier (in or out) and the initial stock price position in relation to the barrier level (up or down). On the other hand, there are four types of external double barrier options, because an external double barrier option is either knocked in or knocked out if the asset price touches the lower or upper barrier.

Heynen and Kat (1994) derived the closed-form valuation formulas for all eight types of external single barrier options. Zhang (1995) obtained one unified pricing formula for external single barrier options. Carr (1995) derived the analytical valuation for the up-and-in call option with external single barrier. Kwok et al. (1998) presented analytic formulas for the valuation of multi-asset options with external single barrier, where the barrier level is exponential. Kwok et al. (1998) used fractional step finite difference scheme for the numerical valuation of barrier options. Chen et al. (2012) derived the closed-form valuation formulas for Parisian external single barrier options. Wong and Kwok (2003) applied the splitting direction technique to derive the pricing formulas of multi-asset options with external double barrier. Banerjee (2003) obtained the closed-form pricing formulas of external double barrier options for the cases where the barriers are monitored during a partial period of the option's lifetime, as well as during the entire period.

The valuation of external barrier options described above has been derived in a Black–Scholes environment. Under a jump diffusion model, Huang and Kou (2006) gave analytical solutions for external single barrier options where the jump sizes have a multivariate asymmetric Laplace distribution. In this paper, we give analytical solutions for external single and double barrier options where the underlying asset prices follow a regime-switching model with finite regimes.

In order to price the external barrier options under the regime-switching model, we need to investigate the joint Laplace transform of the first passage time of one asset value and the value of the other asset. In this paper, we find the joint Laplace transform by using solutions of the matrix quadratic equations. We also provide an iterative algorithm for computing the solutions of matrix quadratic equations with complex parameters. Refer to Asmussen et al. (2004), Breuer (2008), Guo and Zhang (2004), Jiang and Pistorous (2008) and Kim et al. (2015) for the results on the first passage time distribution of a single asset variable under the regime-switching model.

Using the joint Laplace transform of the first passage time of one asset value and the value of the other asset, we present the Laplace transforms of the external barrier option prices under the regime switching model. We also investigate the Greek, delta. We derive the Laplace transforms of the deltas for the external barrier options. We obtain numeric values of option prices and deltas for the external barrier options by using numerical inversion of the Laplace transforms with complex parameters.

The paper is organized as follows. In Section 2, we describe the model in detail. In Section 3, we derive the joint transform of the first passage time of one asset value and the value of the other asset. We also provide an iterative algorithm for computing the numeric values of the joint transform with complex parameters. In Section 4, we derive the transforms of the external single and double barrier option prices. The transforms of the deltas are also derived. Section 5 contains numerical examples. Numerical inversion of the joint Laplace transforms is used to calculate the prices and deltas of external single and double barrier options.

2. The model

We consider a stock price model defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$. Let $S^1(t)$ denote the stock price determining the payoff of the option contract and $S^2(t)$ the stock price which triggers the knock-in or knock-out event. Under the risk neutral valuation framework, we assume that the stock prices $S^l(t)$, $l=1,2$, are given by

$$\frac{dS^l(t)}{S^l(t)} = \left(r_{Z(t)} - q^l_{Z(t)}\right) dt + \sum_{j=1}^{m} \sigma^l_j \mathbb{1}_{\{Z(t)=j\}}\, dW^l_j(t), \quad l = 1, 2. \tag{1}$$

Here, $Z = \{Z(t): t \ge 0\}$ is an irreducible continuous-time Markov process with respect to the filtration $\{\mathcal{F}_t\}$ under the risk-neutral measure $\mathbb{P}$. The Markov process Z has a finite state space $\{1, \dots, m\}$ and the infinitesimal generator G. The processes $W^l_j = \{W^l_j(t): t \ge 0\}$, $l=1, 2$, $j=1, \dots, m$, are Wienner processes with respect to $\{\mathcal{F}_t\}$ under the risk-neutral measure $\mathbb{P}$. Let ρ_j denote the correlation coefficient between W^1_j and W^2_j, i.e., $dW^1_j(t)\, dW^2_j(t) = \rho_j\, dt$, $j=1, \dots, m$. Furthermore, r_j, q^l_j and σ^l_j are real numbers with $\sigma^l_j > 0$, $l=1,2$, $j=1, \dots, m$, and $\mathbb{1}_{\{\cdot\}}$ is the indicator function of the set $\{\cdot\}$.

Let B^+ and B^- denote the external upper and lower barrier variables, respectively, with the following forms:

$$B^+ = \{(t, b^+(t)): t \ge 0\},$$

$$B^- = \{(t, b^-(t)): t \ge 0\},$$

where $b^+(t) = b^+_0 e^{\beta t}$ and $b^-(t) = b^-_0 e^{\beta t}$ with real numbers b^+_0, b^-_0 and β such that $b^-_0 < S^2(0) < b^+_0$. Furthermore, we define