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A rank-dependent utility model of uncertain lifetime ☆

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In a continuous time life cycle model of consumption with an uncertain lifetime, we use a non-parametric specification of rank-dependent utility theory to characterize the preferences of the agent. We prove that time consistency holds for a subclass of probability-weighting function, providing the foundation for a constant rate of time preference that interacts multiplicatively with the hazard rate instead of additively as in the Yaari (1965) seminal model. We calibrate both models to explain the hump in the life-cycle consumption, and show that the multiplicative model is more robust.

A long tradition exists in economics that distinguishes two types of primitive to explain discounting. First, discounting is explained using purely psychological factors, such as impatience, captured by the discount function. If the discount function is exponential, as in the seminal model proposed by Samuelson (1937), then the time preference is characterized by a "pure rate of time preference" (*i.e.* the log derivative of the discount function) that is invariant with time and the level of consumption. Even if some authors considered early on the possibility for the discount factor to be "non-exponential" (for example Yaari, 1964; Harvey, 1986, 1995), only with the behavioral revolution were alternative *ad hoc* parametrical discount functions proposed, and used systematically in the applied economics literature. Among them, the "quasi hyperbolic" discount function (Phelps and Pollak, 1968; Laibson, 1997) is probably the most popular.

The second explanation for discounting is simply to consider that future prospects are uncertain. In this case, considering that the utility of future prospects are weighted according to their probability of being effectively consumed at the given date is reasonable (refer to Sozou, 1998; Dasgupta and Maskin, 2005, for a general discussion of that topic). Among this literature, models of intertemporal choice with uncertain lifetimes are good tools to investigate the theory of discounting. The seminal paper by Yaari (1965) considered expected utility maximizers with known probability distributions of the "age of death", and a standard exponential discounting life-cycle utility. More recent models considered various types of sophisticated utility frameworks to address lifetime uncertainty. For example, Moresi (1999) considered an application of

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the “ordinal certainty equivalent hypothesis” (Selden, 1978). Bommier (2006, 2013) considered a concave transformation of the life-cycle utility to explain “risk aversion with respect to length of life”. Halevy (2008) used the “dual theory of choice” (Yaari, 1987). Ludwig and Zimper (2013), Groneck et al. (2012) and d'Albis and Thibault (2012) considered an ambiguous survival probability.

In this study, we build a model of the intertemporal choice of consumption and saving with an uncertain lifetime in which the agent psychologically transforms her survival probability distribution, such as in the *rank-dependent utility model* (Quiggin, 1982), or in the *cumulative prospect theory* (Tversky and Kahneman, 1992). The idea of introducing a rank-dependent utility in this setting was explored by Drouhin (2001) and Bleichrodt and Eeckhoudt (2006). The originality of this study is that we use continuous time modeling and optimal control to solve the model. With this methodology, we are able to discuss the important topic of time consistency, the main criteria of rationality over time. Following Strotz (1956), there exists a conventional wisdom in economics that considers that any departure from exponential discounting implies time inconsistency. When considering uncertain prospects, part of the literature focuses on the related notion of dynamic consistency (refer to Machina, 1989; Etchart, 2002; Halevy, 2004a,b; Nebout, 2014, for a discussion).

The main result of the study is that any agent who transforms the probability distribution of the age of death with a power function is time consistent. This result provides a foundation for a rate of time preference that interacts multiplicatively with the probability of dying instead of additively in the standard expected utility approach. On empirical grounds, a calibrated version of the model shows that the multiplicative rate of time preference has better properties than the traditional additive rate of the exponential discounting model. This multiplicative rate allows for the solution to certain paradoxes of the literature (the hump of life-cycle consumption, excessive sensitivity to variations in the interest rate).

The remainder of this study is as follows. Section 1 presents the intertemporal utility functional used. Section 2 solves the model in the absence of life annuities. Section 3 discusses time consistency and provides foundations for the multiplicative model of the rate of time preference. Section 4 discusses a calibrated parametrical version of the model on empirical grounds. Section 5 discusses the model when the agent has access to life annuities. Finally Section 6 concludes.

1. A rank-dependent utility model of consumption and savings with an uncertain lifetime

We consider an agent's choice of her consumption profile. A consumption profile is a function of time defined on the interval $[0, T]$, with 0 representing the age of birth and T being an arbitrary constant, interpreted as the maximum possible life duration for the agent. Because we are interested in understanding the manner in which the timing of the decision influences the choice of the consumption profile, we denote the date of the decision by $t \in [0, T)$.

In a first step, we consider the case in which the agent, alive on age t, knows with certainty her age at death, s. We make the following assumptions:

A1 If an agent knows with certainty her age of death s, her intertemporal preferences are represented by an intertemporal utility functional assumed to be additive

$$V_t(c, s) = \int_t^s F(\tau - t) u(c(\tau))\, d\tau \tag{1}$$

with $\lim_{c \to 0} u(c) = +\infty$, $u'(c) > 0$ and $u''(c(\tau)) < 0$.

A2 $\forall \tau \in [t, T], F(\tau - t) > 0$, $F'(\tau - t) \le 0$.

A3 (monotonicity according to lifespan). $\forall c$: $\quad s' > s \Rightarrow V_t(c, s') > V_t(c, s)$.

A1 is the standard assumption for a life cycle model with a certain lifetime. This assumption guarantees the existence of a strictly positive consumption profile throughout the life cycle.

$F(\tau - t)$ is the *riskless discount factor*. When it is strictly decreasing the agent exhibits preference for present consumption.

A1, A2 and A3 together imply that the *per period felicity*, u, and the *intertemporal utility functional*, V, are both necessarily positive in their domain.

A3 indicates that, for a given consumption profile, outcomes are always ranked according to lifespan. When introducing uncertainty, our model is a natural candidate for using a rank-dependent utility.

The agent actually does not know with certainty her age of death.

In the remaining, we will use the letters s or τ for the age of the agent, when it considered as a variable. For some critical ages of the life-cycle, we will use a variation of the letter t (t, is “the age at which the agent plan her future consumption”, T is the “maximum possible age”, t_R will be “the age of retirement”, etc.).

We assume that for a living agent at age t, the age of death $s \ge t$ is an absolutely continuous random variable defined on the interval $[t, T]$. We denote by $\pi_t(s) > 0$ the probability density function of this random variable and $\Pi_t(s)$, the cumulative distribution function. Thus, we have

$$\Pi_t(s) = \int_t^s \pi_t(\tau)\, d\tau$$

with $\Pi_t(T) = 1$.