Computers and Structures 129 (2013) 178–193

Contents lists available at SciVerse ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

An explicit time integration scheme for the analysis of wave propagations

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ARTICLE INFO

Article history:
Received 24 April 2013
Accepted 18 June 2013
Available online 3 August 2013

Keywords:
Wave propagation
Finite elements
Explicit time integration
Desirable numerical dissipation
Numerical dispersion
Bathe implicit time integration

ABSTRACT

A new explicit time integration scheme is presented for the solution of wave propagation problems. The method is designed to have small solution errors in the frequency range that can spatially be represented and to cut out high spurious frequencies. The proposed explicit scheme is second-order accurate for systems with and without damping, even when used with a non-diagonal damping matrix. The stability, accuracy and numerical dispersion are analyzed, and solutions to problems are given that illustrate the performance of the scheme.

1. Introduction

Direct time integration is widely used in finite element solutions of structural dynamics and transient wave propagation problems, and schemes can be categorized into two groups: explicit and implicit methods. A time integration method is implicit if the solution procedure requires the factorization of an 'effective stiffness' matrix and is explicit otherwise [1–3].

In general, each type of integration has its own advantages and disadvantages. Implicit methods require a much larger computational effort per time step when compared with explicit methods. However, implicit methods can be designed to have unconditional stability, in linear analysis, so that the time step size can be selected solely based on the characteristics of the problem to be solved. On the other hand, explicit methods when using a diagonal mass matrix may require only vector calculations. Hence, the computational cost per time step is much lower. However, an explicit method can only be conditionally stable. Therefore, explicit methods may be effective when the time step size required by the stability limit is about the same as the time step size needed to describe the physical problem, and this is frequently the case in wave propagation analyses [1–6].

Accurate finite element solutions of wave propagations are difficult to obtain. Numerical errors due to the spatial and time discretizations resulting in artificial period elongations and amplitude decays, seen as numerical dispersions and dissipations, often render finite element solutions of wave propagation problems to be quite inaccurate [1,6–10]. In particular, large errors in just the few highest frequency modes contained in the mesh shown as spurious oscillations can severely impair the accuracy of the solution. These spurious oscillations may increase in time since the dispersion and dissipation errors accumulate as the waves propagate.

Much research effort, following different approaches, has been focused on reducing the dispersion and dissipation errors. Of course, to reduce the errors from the spatial discretization, higher-order spatial discretizations can be employed [11–16]. However, the use of high-order elements can be computationally expensive and may not have the generality as does the use of the traditional finite element procedures employing low-order elements. Linear combinations of consistent and lumped mass matrices [17–21] or modified spatial integration rules for evaluations of mass and stiffness matrices [22–24] may also be used to obtain better solution accuracy. However, these schemes are different from those commonly used in structural dynamics and do not lead to a general solution procedure. Errors due to the spurious oscillations can also be reduced by the use of filtering [24–26] for specific points in space and time. These schemes can be valuable to obtain improved solutions for a number of spatial and time points but in engineering practice, accurate solutions are generally sought over the complete problem geometry and all time considered.

Many direct time integration schemes introduce numerical dissipation to improve the solution by suppressing the high frequency spurious modes [1,2,27,28]. However, it is difficult to obtain an effective scheme, since the numerical dissipation should be large

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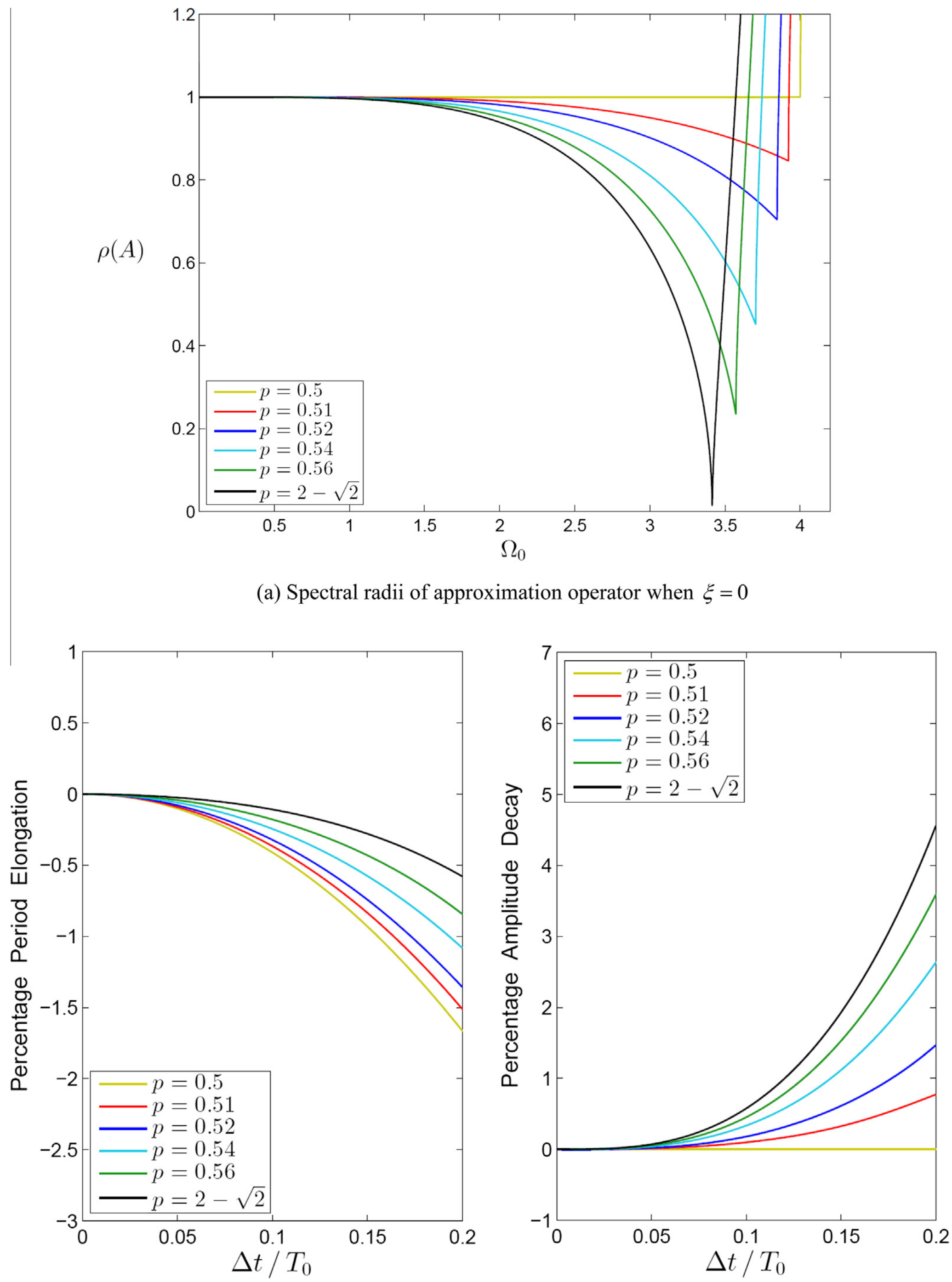
http://dx.doi.org/10.1016/j.compstruc.2013.06.007

enough to suppress the high frequency spurious modes, while at the same time keeping good accuracy in the low frequency modes. The search for an effective such scheme is very important since, in engineering practice, such method could be used for structural dynamics and wave propagation problems in a uniform manner.

Among implicit methods, the Bathe method [29–31] has been shown to result in remarkably accurate solutions by suppressing the high frequency spurious modes [6]. The property of this scheme to 'cut out' high frequency modes that cannot be spatially resolved and to integrate those modes accurately that can be spatially resolved results into relatively small dispersion error [6,32].

Considering explicit methods, the central difference method is still a widely used scheme. It has the largest time step stability limit of any second-order accurate explicit method [33,34]. However, the central difference method requires a matrix factorization for systems with a non-diagonal damping matrix, a shortcoming that has been addressed, see e.g. Refs. [35,36], and in particular, since the method is a non-dissipative scheme, the solution accuracy can be severely ruined by the dispersion errors in the high frequency modes.

The development of dissipative explicit methods has been much pursued [37]. Schemes have been presented by Newmark [38],



(a) Spectral radii of approximation operator when $\xi = 0$

(b) Percentage period elongations and amplitude decays

Fig. 1. Proposed scheme for various values of *p*.