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Uncertain dynamics, correlation effects, and robust investment decisions

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ABSTRACT

We analyze a firm's investment problem when the dynamics of project value and investment cost are uncertain. We provide an explicit solution using a robust method for an ambiguity averse firm taking this into account. Ambiguity aversion regarding a common risk factor impacts differently than ambiguity aversion regarding investment cost residual risk. Correlation between project value and investment cost matters; ambiguity aversion regarding common risk can decrease the investment probability only if correlation is positive. Ambiguity aversion regarding residual risk always increases the investment probability. When only project value is risky, volatility can monotonically decrease the investment threshold; this does not hold with the multiple prior method.

1. Introduction

Corporate investments play a central role for any firm. A key problem is when to initiate a given investment project. The firm's perception of the project's investment cost and returned value is crucial for the decision, but the literature often neglects that the dynamics of the investment cost and the dynamics of the project value are subject to uncertainty per se. Our paper considers how uncertainty—in the sense of imperfect knowledge about parameters of the underlying processes—impacts the firm's investment policy, when the project value and the investment cost have a common risk factor and there is additional investment cost residual risk.

A firm which takes this kind of uncertainty into account is said to be ambiguity averse. We show that ambiguity aversion has a substantial impact on the firm's investment decision. On one hand, more ambiguity aversion decreases the value of the option to wait. That is, the investment threshold (the project value at investment relative to the investment cost) decreases. In fact, the value of the option to wait can be completely eliminated by ambiguity aversion. On the other hand, ambiguity aversion does not unambiguously increase the investment probability. The resulting effect depends on whether ambiguity aversion regarding the common risk or ambiguity aversion regarding the investment cost residual risk dominates. It also depends on the time horizon considered and the correlation between the two dynamics. The volatility of project value traditionally increases the investment threshold, but this effect is strongly reduced by ambiguity aversion. This reducing

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effect implies that the correlation between the two dynamics plays a central role for the investment probability. Indeed, ambiguity aversion regarding common risk can decrease the investment probability, but only if correlation is positive. Ambiguity aversion regarding residual risk always increases the investment probability.

Our paper is related to, for example, Nishimura and Ozaki (2007), Trojanowska and Kort (2010), and Miao and Wang (2011) who use an alternative approach—the multiple prior approach—to address ambiguity aversion. They also find that the investment threshold decreases in ambiguity aversion. However, unlike these papers the present paper handles a multi-dimensional setting and we can analyze the effect of a common risk factor—and hence correlation—between the project value and the investment cost. As discussed earlier, correlation plays an important role in the investment problem. Furthermore, if only the project value changes over time, the investment threshold can be non-monotonic with respect to project value volatility in the multiple prior setting. In contrast, our setting provides the more intuitive result that the investment threshold is monotonic in project value volatility. Our setting also puts more weight on the firm's best guess of the parameters in the underlying dynamics. We illustrate that this leads to a substantial different investment probability than if the multiple prior approach is used.

More generally, the firm's investment problem has been studied extensively in the literature. Since McDonald and Siegel (1986), a common approach to analyze this problem has been to use real options analysis. This approach has found widespread applications in corporate finance.[1] The starting point of the analysis is to specify a model describing the dynamics of the project value and the investment cost. Usually, the correctness of the specified dynamics is not considered as part of the analysis. One approach to deal with this is to relax the assumption of complete information of the parameters by considering incomplete information models with updating of beliefs (e.g., Decamps et al., 2005).[2] A different approach considers a decision maker—i.e., a firm—that does not fully trust the model used in the analysis, and the firm therefore takes alternative models into account. This behavior is illustrated by the Ellsberg paradox (Ellsberg, 1961), and the setting is known as Knightian uncertainty or ambiguity. That is, the firm does not trust the probability measure employed in the model, i.e., the firm is ambiguity averse.

To model ambiguity aversion, we use the so-called robust decision method by Hansen and Sargent (2001).[3] The firm uses a reference model as its starting point, but worries that the model does not correctly describe the setting. Hence, the firm fears that the project value and the investment cost can evolve very differently from what is predicted. Therefore, the firm considers deviations from the reference model. The considered deviations depend on the perceived worst outcomes as well as a penalty for deviating from the reference model. This implies that the parameters affected by ambiguity are chosen endogenously. Anderson et al. (2003) extend the robust method to a continuous-time framework with a robust Hamilton–Jacobi–Bellman (HJB) equation. Using this we end up with a partial differential equation that is significantly different from the usual HJB equation in real options analysis. Therefore, a central result in our paper is that we can derive an explicit solution and that we can use this to derive the value of the option to invest together with the optimal investment threshold.

We set up the general model in Section 2 in which we derive explicit formulas for the value of the option to invest and the investment threshold. The model is further analyzed using numerical analysis in Section 3. In Section 4 we compare our results with those from the multiple prior method. Finally, Section 5 concludes and all proofs are postponed to appendices.

2. The robust decision to invest

We consider a firm with a perpetual option to invest in a project. If the firm pays the irreversible investment cost I, it receives a project with value V in return. What complicates matters for the firm is the fact that there is uncertainty about the probability law driving the evolution of the project value and the investment cost. If the firm takes this uncertainty into account, it is ambiguity averse.

2.1. The ambiguous investment problem

Let $(\Omega, \mathcal{F})$ be a measurable space and assume that the firm uses the probability measure $\mathbb{P}$ as a reference probability measure. This is the measure that an ambiguity neutral firm would apply. Let $\boldsymbol{B} = (B^1, B^2)^\top$ be a two-dimensional Brownian motion with respect to $\mathbb{P}$ and a filtration $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$. At time t the process for the project's value and the investment cost have the dynamics:

$$dV_t = V_t\left(\mu_V\, dt + \sigma_V\, dB_t^1\right), \tag{1}$$

[1] A standard reference for this approach is found in, for instance, Dixit and Pindyck (1994). Other applications than the firm's investment problem are, for example, capital structure (Brennan and Schwartz, 1978; Hackbarth et al., 2007), mergers and acquisitions (Lambrecht and Myers, 2007; Morellec and Zhdanov, 2008), R&D (Berk et al., 2004), and corporate risk management (Boyle and Guthrie, 2006).

[2] Decamps et al. (2005) consider incomplete information about the drift of the project value. Their approach requires infinitely many data to sufficiently reduce the variance of the parameter. Consequently, the optimal investment decision depends on time as well as on the underlying state variables (see e.g., Anderson et al., 2003). Furthermore, the firm prefers the project with an unknown drift compared to the project where the prior mean is the fixed drift.

[3] See Hansen and Sargent (2008) for an introduction to robust methods. Ambiguity aversion is also used in many other areas in finance and economics, e.g., Chen and Epstein (2002), Maenhout (2004, 2006), Caskey (2009) and Garlappi et al. (2013).