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Two shape parametrizations for structural optimization of triangular shells

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ABSTRACT

The complexity of state-of-the-art tools for shell optimization may limit their applicability in common practice. We propose two shape parametrizations, inserted into a robust and simple procedure, based on linear finite elements and gradient-based optimization. We represent the mid-surface by triangular Bézier surface and *ad-hoc* heuristic functions. The first method allows searching for a general shape, while in the second one the functions are chosen according to structural and aesthetical criteria. Small number of design variables ensures efficiency. The procedure is applied to Kresge auditorium at MIT. Both parametrizations provide satisfactory results, with slightly better performances of Bézier surface representation.

1. Introduction

A shell is a thin-walled structure whose load bearing capability is almost entirely provided by its shape. The design methodology that leads to the final shape of shells has experienced a remarkable change over the time. In the past centuries, the design methodology was mainly empirical and based on the tradition, culture, experience, and knowledge of the designer. Prototypes of the modern reinforced-concrete (RC) shells, with larger thickness-to-curvature radius ratios (s/R), are the domes built since the Antiquity to the Renaissance. Prominent examples for their spans (unsurpassed until the introduction of reinforcements) are the concrete dome of the Pantheon in Rome (2nd century AD, $s/R = 1/3.7 \div 1/18$) and the double masonry shell of Brunelleschi Dome in Florence (15th century, $s/R = 1/8.5$, considering the total thickness including the interspace). From the end of 19th century to the 1960s, impressive progresses in the conceptual design of shells were made by master builders like F. Dischinger (1887–1953), P.L. Nervi (1891–1979), E. Torroja (1899–1961), A. Tedesko (1903–1994), F. Candela (1910–1997), to mention a few. Their outstanding structures were the result of a deep understanding of the interrelation between geometry and statics, the former assigned *a priori* and often described by analytical functions [1–3].

If the shape of the shell is not assigned *a priori*, an inverse problem arises, consisting in finding the geometry corresponding to a membrane-dominated stress-state under assigned loads [4,5]. To this aim, two main experimental methods were used: (i) the hanging model and (ii) the pre-stressed soap-film analogy. These two methods are usually referred to as 'form finding' methods [3].

The hanging-model method consists in suspending one- or two-dimensional objects with no bending and compressive stiffness (a cable, a cable net, or a fabric), so that they undergo a purely tensile stress-state. When the desired shape is reached, the geometry is 'frozen' and inverted, so that a bending-free, compressive stress-state is obtained. For the plane problem of one-dimensional hanging cables, the method is known as 'catenary method' and was probably already used in the design of ancient structures, such as the Arch of Taq-i Kisra in Ctesiphon (6th century AD). The mathematical description of the corresponding catenary curve was firstly studied by Robert Hooke (1635–1703), and applied to a number of cases, such as the design of St. Paul's Cathedral in London by C. Wren (1632–1723) and the strengthening by G. Poleni (1673–1781) of St.-Peter's Dome in Rome. The three-dimensional case of hanging chain-nets might have been adopted already in Middle Ages in the design of some gothic churches, as conjectured in [6], and was systematically used by A. Gaudì (1852–1926) [7–9]. Since the 1960s, the principle of hanging models has been intensively used by H. Isler (1926–2009) to design elegant ultra-thin RC shell structures [10], with spans up to 40 m and s/R as low as 1/1000.

The soap-film method is used for form-finding of tensile structures or very shallow shells, for which the normal vector to the surface is almost vertical, i.e. the difference between the uniform vertical load and a surface pressure is negligible [11].

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The extraordinary reliability and long-term performances of shell structures designed through the two form-finding methods prove their efficiency. Nevertheless, they present some limitations, such as the impossibility of accounting for multiple load conditions, variable thickness, and other not properly structural aspects.

Nowadays, form-finding physical approaches are replaced by numerical simulations through the finite element (FE) method, mostly by using shell elements [12]. However, these approaches may present numerical issues, which justify the large research effort of the last decades. In the hanging method the isotropy/anisotropy of the material and the value of its elastic moduli affect the results; the design variables are not obviously defined; and the solution may be not unique. The numerical translation of the soap-film analogy leads to system matrix singularities, when nodal displacements occur in the plane tangential to the shell mid-surface [13]; this problem can be circumvented by the updated reference strategy [13,14], whose capability of facing incompatible stress states is discussed in [15,16]. Finally, both form-finding methods cannot consider, beside the shape, other structural (optimized variable thickness, stability, modal properties, etc.) and non-structural (e.g., costs) aspects.

These drawbacks are overcome by the application of structural-optimization methods [13,17–19]. Structural optimization offers a powerful tool to handle complicated problems and it has been applied to a larger variety of structural typologies. The initial numerical applications dealt with latticed and framed structures, modelled through truss and beam elements. For a review of the methods developed between 1960s and 1980s see [20]. For shell structures, a comparative review of optimization methods, including their relationships with form-finding methods, is given in [3,16]. Incidentally, also the physical intuition of the form-finding methods contains the concept of optimization, in that the shape of a hanging model or of a soap-film corresponds to a minimum of the total potential energy of the associated mechanical system. Therefore, the optimization methods can be regarded to as a generalization of the form-finding ones.

The first studies addressing the optimum shape of a structure were performed through ad-hoc computer codes [21,22]. More recently, the solution of the structural part has been achieved by commercial software [23–26] and the combination of structural FE solvers with commercial optimization packages results in an efficient tool [27]. Some advanced FE programs also integrate an optimizer [28,29].

A structural optimization problem is made up of three parts: (1) the geometrical part, where the shape of the shell is parametrically described, providing the design variables; (2) the mechanical part, where the most suitable objective function and possible constraints are chosen; and (3) the mathematical part, where the minimization problem is stated.

The present work uses structural-optimization methods and focuses on the first part, i.e. the geometrical description of the geometry.

We study the well-known Kresge auditorium of MIT, a triangular shell stiffened by edge arches, for which the benefits of shape optimization can be easily expected. The shell has in fact a spherical geometry and would undergo significant bending stresses, if the three stiffening arches were removed, as we do in the optimization process. The Kresge auditorium is widely studied in the literature, so that a comparison of the optimization results can be made.

We use a methodology based on structural optimization by representing the geometry of the mid-surface through two different approaches.

The first one is completely general and allows to search for a large variety of shapes. It consists in an original application of Bézier surface, where triangular patches are used instead of the more common quadrilateral ones [3,13].

The second one is a case-dependent approach that makes use of specific parametric functions, chosen to mitigate the bending stresses arising at the shell boundaries once the edge arches are removed. These functions are also chosen to satisfy some aesthetic criteria, a task which would be more difficult to achieve by the previous, fully free-form approach. The use of *ad-hoc* heuristic functions has the further advantage to control the shape through a very low number of design variables.

The mechanical model is treated by standard linear-elastic FE analysis. The optimization problem is stated in a classical form and numerically solved by a constrained gradient-based minimization algorithm. Over the last decades, new optimization algorithms based on stochastic search methods have been developed, such as Differential Evolution (DE) algorithms, Genetic Algorithms (GA), and Particle Swarm Optimization (PSO) algorithms (for a review see [30]). The efficiency of these algorithms for structural optimization problems is investigated in [31–37].

For problems involving a large number of design variables, evolutionary algorithms are generally more suitable than gradient-based methods (e.g., gradient information and sensitivity analysis are not required), while the latter may guarantee a better convergence rate for a reduced number of design variables. In this work, the gradient-based method turns out to be suitable for our optimization problem.

The overall procedure is based on simple tools (linear static analysis and a simple optimization algorithm). Moreover, especially in the case in which *ad-hoc* functions are used to describe the geometry, the number of design variables is reduced. In both cases, the resulting shape is very similar to the one obtained through more sophisticated methods, which are not promptly available to average designers in the engineering practice.

This work is organized as follows: in Section 2, the structural optimization problem is formulated and the two surface representations are introduced. In Section 3, we apply the optimization procedure to the Kresge auditorium at MIT. Results and comparisons with the initial spherical shape are presented in Section 4. Finally, the main conclusions are drawn in Section 5.

2. Structural optimization

The general formulation of the problem is

$$\begin{aligned} &\underset{s\in D}{\text{minimize}} && f(s) \\ &\text{subjected to} && g_i^{\text{eq}}(s) = 0, \quad i = 1,\dots,N_{\text{eq}}, \\ & && g_j^{\text{in}}(s) \leqslant 0, \quad j = 1,\dots,N_{\text{in}}, \end{aligned} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is the objective function, $s \in \mathbb{R}^n$ is the n-dimensional design variable vector, g_i^{eq} and g_j^{in} are the ith equality and jth inequality constraints.

The choice of the objective function depends on the purpose of the optimization procedure: some examples are the cost, the reliability, the stiffness, the weight, the first fundamental period, the so-called 'volumetric displacement' (i.e., the integral over the surface of the product of thickness and displacements, [28]), the stress levelling. In our case, the aim of the optimization is a bending-free state of stresses. This is achieved by minimizing the strain energy, because it is largely due to flexural behavior (e.g., [28]).

For a general structural optimization problem, the design variables can be geometry parameters (e.g. variables controlling the shape), structural properties (e.g. cross-section area, thickness, inertia moment), topological parameters (e.g., presence of holes controlled by an element density function in [0,1] as in [27]), and the material properties. In the case of shell optimization