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Large displacement analysis of sandwich plates and shells with symmetric/asymmetric lamination

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ABSTRACT

This paper proposes a kinematic model for sandwich plates and shells, utilising a novel zigzag function that is effective for symmetric and asymmetric cross-sections, and employing a piecewise through-thickness distribution of the transverse shear strain. The proposed model is extended to large displacement analysis using a co-rotational framework, where a 2D local shell system is proposed for the direct coupling of additional zigzag parameters. A 9-noded co-rotational shell element is developed based on the proposed approach, which utilises the MITC method for overcoming locking effects. Several linear/nonlinear analysis examples of sandwich structures demonstrate the effectiveness of the proposed approach.

1. Introduction

Sandwich structures consisting of two stiff outer layers bonded to a soft core have been widely adopted in many engineering applications. Due to the large face-to-core stiffness ratio, such structures are characterised by a zigzag form of displacements. The classical lamination theory (CLT) and the first-order shear deformation theory (FSDT) [1], representing extensions of the Kirchhoff and Reissner–Mindlin plate theories to the laminate, cannot accurately predict the response of sandwich structures due to the assumption of linear variation in displacement over the thickness. Higher-order shear deformation theories (HSDTs) [2,3] improve the accuracy of the global response by introducing additional variables with higher-order out-of-plane *z* expansions of the displacement fields, but these *z* expansions, which are defined at the multi-layer level, cannot describe the discontinuity associated with the variation of mechanical properties through the thickness.

There are two main approaches that include the zigzag effect into 2D modelling: layer-wise (LW) description and equivalent single layer (ESL) description with the inclusion of Murakami's zigzag function [4]. LW models [5–7] regard each layer as an independent plate or shell, and employ any of the CLT, FSDT and HSDTs at the layer level. The compatibility conditions are satisfied by imposing displacement constraints at laminar interfaces. Nevertheless, the number of displacement variables in LW models depends on the number of constitutive layers, though the number of displacement fields can be reduced by also enforcing the continuity of transverse stresses at laminar interfaces [8–14]. These methods are typically labelled as zigzag theories.

The ESL description considers the zigzag effect with relative ease, where a piecewise linear zigzag function, such as the one first proposed by Murakami [10], is added to the FSDT and HSDTs [11,12]. Approaches in this category are denoted as 'EDZ' models (where 'E' stands for the ESL description, 'D' stands for the employment of principle of virtual displacements, and 'Z' indicates the inclusion of a zigzag function) [11]. The EDZ models improve the results of FSDT and HSDTs with relative ease, and the degrees of freedom (DOFs) of EDZ models are independent of the number of layers. On the basis of the EDZ models, a group of mixed formulations have also been developed, denoted as 'EMZC' models (where 'M' stands for the mixed formulation, and 'C' means the fulfilment of inter-laminar continuity) [11], where continuous transverse shear and normal stresses are assumed and Reissner's variational principle is employed, thereby achieving the continuity of both displacements and transverse stresses. The EDZ and EMZC models have been widely used in the analysis of thin-to-thick laminations as well as sandwich structures [11–15]. It is noted, however, that the effectiveness of the EDZ and EMZC models relies on the effectiveness of Murakami's zigzag function (MZZF), which depends further on the material properties and thickness of each constitutive layer as well as the stacking sequence. In plate bending problems associated with an asymmetrically laminated sandwich plate [15], it has been shown that higher-order *z* expansions of

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displacements are required for the EDZ models to achieve sufficient accuracy owing to the ineffectiveness of the MZZF, while Carrera [16] suggested that the effectiveness of the MZZF may be improved with the employment of the mixed assumption. Nevertheless, the ESL models with the inclusion of MZZF provide a convenient approach for considering the lamination effects in terms of accuracy versus the required computational effort, and these models have also been employed in finite element formulations to analyse sandwich and lamination problems involving geometric nonlinearity [17–20].

Motivated by Murakami's work [10], this paper proposes an efficient three-layered model for the analysis of sandwich plates and shells. While similar in principle to the EDZ model, an important difference is the introduction of a novel zigzag function over the full plate thickness that is specific to sandwich plates/shells with a soft core. This enriches the classical Reissner–Mindlin formulation [21,22] by allowing for cross-sectional warping, and satisfies the continuity of displacements at laminar-interfaces a priori via the assumed zigzag mode. The proposed zigzag function is shown to provide good accuracy for both symmetric and asymmetric lay-ups, while achieving computational efficiency through the use of a minimal number of additional zigzag displacement fields. On the other hand, a piecewise linear–constant–linear distribution is assumed for the transverse shear strain, which imposes no constraints on transverse shear stresses but is shown to provide an accurate representation of the actual distribution without sacrificing computational efficiency.

In formulating large displacement 2D shell elements for small strain problems, the relationship between the strain and displacement fields is highly nonlinear and complex if the displacement fields are referred to a fixed coordinate system, such as in the Total Lagrangian approach [23,24], where the nonlinear strain terms arise mainly from the element rigid body rotations. Instead, the co-rotational approach, which decomposes the element motion into rigid body and strain-inducing parts via the use of a local co-rotational system, allows the employment of low-order, even linear, relationships between the strain and local displacement fields [25]. In this respect, the co-rotational approach shifts the focus of geometric nonlinearity from the continuum level to the discrete nodal level, and it can act as a standard harness around different local element formulations [26], upgrading such formulations to large displacement analysis with relative ease. In addition, when laminations are considered, the co-rotational system provides a reference orientation for the definition of the zigzag displacement variables which are associated with local cross-sectional warping only. In this context, co-rotational transformations of the zigzag displacement variables are avoided in this work through the introduction of a 2D 'shell' coordinate system, which follows the local co-rotational element system, thus achieving significant computational benefits. Further benefits arise in the local element formulation with the definition of a 2D 'shell' coordinate system that is continuous over the shell structure, where three such definitions are proposed in this paper. Notwithstanding the above benefits, it is important to note that the co-rotational approach offers no particular benefits in large strain problems, for which a Total Lagrangian approach would be more suited [24].

The application of the proposed sandwich shell model is illustrated for a 9-noded shell element [27,28], which employs a bisector co-rotational system [25] for modelling geometric nonlinearity. The basic local displacement variables consistent with the Reissner–Mindlin formulation are related to the global variables according to discrete nonlinear co-rotational transformations, while the additional zigzag displacement variables are defined directly in the 2D curvilinear shell system. Furthermore, in order to alleviate membrane and shear locking which arise with conforming displacement-based shell elements, an assumed strain approach is considered. Amongst the different assumed strain methods, the Mixed Interpolation Tensorial Components (MITC) method [29,30] is widely used to overcome locking, offering a two-level approximation that samples and interpolates strain components in a covariant coordinate system at a selection of positions. The application of the MITC method to a 9-noded shell element has been shown to yield a much improved element performance [31–33], thus the MITC approach is utilised herein for each constitutive layer of the 9-noded shell element to overcome locking.

The paper proceeds with presenting the proposed kinematic description for sandwich shells, the effectiveness of which is then demonstrate with reference to a 1D linear problem. The enhancements required for large displacement analysis of shells are subsequently presented, and the application of the proposed sandwich shell model is illustrated for a 9-noded co-rotational shell element. Several linear and nonlinear numerical examples are finally presented to demonstrate the accuracy and efficiency of the proposed approach for the analysis of sandwich shell structures with both symmetric and asymmetric lay-ups.

2. Kinematic description for sandwich shells

A kinematic model is proposed in this section for sandwich shells, with specific reference to the through-thickness variation of displacement fields and the transverse shear strains. Fig. 1 depicts the sandwich model for a plate along with the local coordinates, where the *x*- and *y*-axes are located at the middle surface while the *z*-axis is normal to the plate, and where each layer is identified by a unique index. It is important to note that while the kinematic descriptions is initially presented for a plate problem, it is equally applicable to local formulations of shallow shells, as elaborate in Section 2.2. Furthermore, through incorporation within a co-rotational framework, it is also applicable to the nonlinear analysis of general curved shells, as presented in Section 4 and demonstrated by the numerical examples of Section 6.

2.1. Geometry and displacement fields

In this sandwich plate model, a piecewise linear variation of planar displacements in the *z* direction is assumed, thus readily satisfying C^0-continuity at laminar interfaces. Accordingly, the through-thickness distribution of the planar displacements can be decomposed into four independent displacement modes $\Lambda_i(z)$ $(i = 1 \to 4)$ (Fig. 2), including a constant and a linear mode, Λ_1 and Λ_2, in accordance with the Reissner–Mindlin kinematic hypothesis, as well as two zigzag modes, Λ_3 and Λ_4, accounting for the zigzag effect. Λ_3 and Λ_4 are both orthogonal to the constant and linear modes while associated with respectively different and identical rotations of the normal in the two face sheets; these are expressed as:

$$\Lambda_3(z) = \begin{cases} \hat{a}_3^{(1)} z + \hat{b}_3^{(1)} & z \in [h_{1-}, h_{1+}] \\ \hat{a}_3^{(2)} z + \hat{b}_3^{(2)} & z \in [h_{2-}, h_{2+}] \\ \hat{a}_3^{(3)} z + \hat{b}_3^{(3)} & z \in [h_{3-}, h_{3+}] \end{cases} \tag{1}$$

$$\Lambda_4(z) = \begin{cases} \hat{a}_4^{(1)} z + \hat{b}_4^{(1)} & z \in [h_{1-}, h_{1+}] \\ \hat{a}_4^{(2)} z + \hat{b}_4^{(2)} & z \in [h_{2-}, h_{2+}] \\ \hat{a}_4^{(3)} z + \hat{b}_4^{(3)} & z \in [h_{3-}, h_{3+}] \end{cases} \tag{2}$$

in which h_- and h_+ denote the values of *z* at the bottom and top of the cross-section, respectively; h_{k-} and h_{k+} refer to the values of *z* at the bottom and top of layer (*k*), respectively; and expressions of $\hat{a}_i^{(k)}$ and $\hat{b}_i^{(k)}$ $(i = 3, 4)$ are provided in Appendix A.