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Network structure, games, and agent dynamics

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ABSTRACT

Consider a group of agents embedded in a network, repeatedly playing a game with their neighbors. Each agent acts locally but through the links of the network local decisions percolate to the entire population. Past research shows that such a system converges either to an absorbing state (a fixed distribution of actions that once attained does not change) or to an absorbing set (a set of action distributions that may cycle in finite populations or behave chaotically in unbounded populations). In many network games, however, it is uncertain which situation emerges. In this paper I identify two fundamental network characteristics, boundary consistency and neighborhood overlap, that determine the outcome of all symmetric, binary-choice, network games. In quasi-consistent networks these games converge to an absorbing state regardless of the initial distribution of actions, and the degree to which neighborhoods overlap impacts the number and composition of those absorbing states.

1. Introduction

Sometimes our decisions influence and are influenced by others and game theory has proven to be a powerful tool for modeling such situations. Likewise many of our economic decisions are channeled through a few specific individuals or specific institutions with whom we interact on a regular basis and networks are a useful way to map such relationships. The combination of the two, embedding games into networks, allows us to investigate how different types of economic organizations (different networks) might affect interdependent decisions made under a variety of economic situations (different games). For this reason the theoretical literature on networks games is especially deep with contributions coming from physics, biology, sociology, and economics.

Much of what we know about the evolution of local decision making and its percolation through a population comes from analyzing particular games played on particular networks. To mention only a few, Axelrod (1984) and Nowak and May (1992, 1993), investigate prisoners' dilemma games played on a grid, while Eshel et al. (1998) model that game on a ring. They find that the shape of some networks allows cooperation or altruism to survive in a prisoners' dilemma game, suggesting there may be a spatial explanation as to why cooperation survives in the wild. Bramoullé and Kranton (forthcoming) investigate local public goods and networks and find that social network structure influences the agents' investment in information and their willingness to share. Ellison (1993) and Young (1998, 2002) explore games of

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coordination on grids and rings demonstrating that in most situations, the population eventually settles on one of the two conformist equilibria.

Reviewing much of the network game literature, Goyal (2007) comments that networks are rich and complicated objects that make it "difficult to obtain tight and general predictions regarding their effects on individual behavior" (p. 54). However, he proceeds to generalize games of coordination, showing that in any connected network, a social coordination game with a best response decision rule and random perturbations of actions will eventually converge to a Nash equilibrium. In another comprehensive review of games on graphs Szabo and Fath (2007) also note that network effects have been investigated for only limited games and network structures and that recent research (such as Hauert et al., 2006) is trying to generalize across more games. In their conclusion they write, "We hope that future efforts will reveal the detailed relationships between these internal properties" (Szabo and Fath, 2007, p. 196).

This paper continues that search to find general relationships that tell us about agent dynamics in network games. We identify two fundamental network characteristics that influence the distribution of actions in all symmetric, binary-choice games played on any network using imitation-type updating rules. In short, the neighborhood configuration of a group's boundary determines whether a game converges to a stable distribution of actions or alternatively enters a cyclical pattern of decisions. Neighborhood configurations also affect the potential number of those distributions and the mix of actions within each. To ensure convergence to an absorbing state, a network must have quasi-consistent boundaries (defined below). This is a strong result. Any binary-choice game played on any network with quasi-consistent boundaries will converge to a fixed state of actions—any binary-choice game, any set of payoffs, any initial distribution of actions, all the time. And the opposite is also demonstrated: in a network lacking this boundary characteristic, there is always some distribution of strategies and payoff combinations that will trigger cycles of behavior.

Beyond the dynamics of game play there is parallel interest in the relationship between network structure and the composition of that absorbing set or state of actions. To that end we show that neighborhood overlap, the extent to which neighborhoods have common neighbors, affects the distributions of actions taken by players and it also affects the number of payoff combinations that trigger a phase transition, the sudden adjustment from one distribution of decisions to another. To my knowledge no one has identified network structures so fundamental that they apply to all binary-choice games and all networks.

Knowing more about the pivotal role that organizational structure plays in interdependent decision making can help us more deeply understand many economic institutions. For example, supply chains, Boards of Directors, managements' organizational chart of their firm, the committee structure in Congress, firms adopting similar or competing technologies, bureaucracies, sports leagues, street gangs, and other such social and economic organizations involve agents interacting within an organization making decisions whose consequences are related to the decisions of others. Recognizing the degree to which these networks possesses the boundary characteristics discussed below can help us predict aggregate evolutionary behavior, construct testable hypotheses about that behavior, and/or illuminate the actions that can nudge a group to a particular result.

Section 2 formally defines the games, networks, and decision making strategies considered in this paper and Section 3 presents some exploratory virtual experiments that guide the analytical study in the remaining sections. Section 4 addresses the dynamics of play examining how network structure affects convergence to absorbing sets or states while Section 5 examines the composition of those sets or states. Section 6 broadens the applicability of the results, and Section 7 concludes.

2. Network games and decision making

2.1. Games on networks

Symmetric binary-choice games can be represented with the familiar payoff matrix in which players select an action, A or B, to receive payoffs a, b, c or d.

	A	B
A	a, a	b, c
B	c, b	d, d

An ordering of payoffs creates a particular game. For example, $a>b>c>d$ establishes a game in which decision A becomes the dominant decision for both players, resulting in the play "AA" and payoffs of "a" for each. Changing the relative magnitudes of these payoffs creates other games; for example, $c>a>d>b$ defines a prisoners' dilemma game in which A is the cooperative choice and B is defection. All possible combinations yield $4!=24$ different orderings, half of which are mirror images of other games.[1] We study all of these games.[2]

[1] For example payoffs $d>c>b>a$ mirrors the first example with BB being the dominant equilibrium.

[2] Rapoport (1966) provides a complete taxonomy of binary-choice games including asymmetric payoffs. However, incorporating asymmetric games into networks requires an *a priori* assignment of who is player #1 and player #2 for each pairing. This artificial assignment is avoided by focusing on symmetric games.