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Formulation of the dynamic stiffness of a cross-ply laminated circular cylindrical shell subjected to distributed loads

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ABSTRACT

This paper describes a procedure for taking into account distributed loads in the calculation of the harmonic response of a cross-ply laminated circular cylindrical shell using the dynamic stiffness method. This work is a direct continuation of a previous work concerning isotropic materials. Equivalent loads are established on element boundaries to determine the response of the system. Therefore, the vibration analysis is solved with numerical examples in order to determine the performances of this approach. The method allows reducing both the size of the model and computing time, and ensures higher precision compared to the finite element method.

1. Introduction

Laminated composite structures is the subject of much going research since the middle of 80s. Nowadays, such structures are frequently used in various engineering applications in the aerospace, mechanical, marine, and automotive industries. Qatu et al. [1] have recently published a review on the dynamic analysis of composite shell structures. General theoretical developments can be found in the textbook written by Reddy [2] and specific works concerning the dynamic analysis of such structures are presented in many papers. These works can be classified into five main subjects of interest regarding geometrical or material considerations. Thus, these works concern:

- multilayered four edges panels [3–16],
- multilayered non-cylindrical shells of revolution as conical shells, spherical shells and circular plates [17–21],
- multilayered non-circular cylindrical shell [22–27],
- influence of geometrical or material irregularities on the dynamic behaviour of multilayered circular cylindrical shells [28–33],
- multilayered circular cylindrical shell.

The last subject concerns our paper and a more refined description is required. Three main subjects can be defined. The first one concerns the development of improved shell theories. For example, Khdeir et al. [34] have described a modification of the Sanders' shell theory that takes into account a parabolic distribution of the transverse shear strains to deal with the transient response of circular cylindrical shells. Soldatos and Timarci [35,36] have presented an unification of the classical Donnel-, Love- and Sanders-type shell theories. More recently, Jin et al. have studied such unification as well [37]. Tarn [38] derives a two-dimensional theory with shear effect using an asymptotic expansion in three-dimensional elasticity. By using the method of power series, Matsunaga [39] has derived a higher-order theory that allows to take into account transverse shear, normal strains and rotatory inertia. The two other subjects concern the development of numerical approaches for solving the problem of vibrations of circular cylindrical laminated shells.

On one hand, specific formulations of the Finite Element Method [40] or Rayleigh–Ritz procedures [41,42] are described [43,44]. However, these methods have limitations and may require a large amount of computer time, especially when the frequency band widens, leading to a large system of equations. These methods are often restricted to low frequency applications.

In the other hand, analytical or semi-analytical methods are developed to find alternative methods in order to reduce computing times and computer storage requirements while trying to enhance the accuracy of the results over larger frequency ranges.

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The work presented in this paper is part of this trend. For example, Nosier and Reddy derived an analytical approach based on Levy-series type solution [45]. Zhang extended the wave propagation approach to calculate the natural frequencies of such structures [46]. Haftchenari et al. [47], Alibeigloo [48] studied the dynamic behaviour of composite cylindrical shells using the Differential Quadrature Method. More specifically, the latter was used to investigate the effect of edge conditions on the vibrations of anisotropic laminated cylindrical shells. The Dynamic Stiffness Approach is used by Chronopoulos et al. for analysing the harmonic response of curved and cylindrical shells [49]. Fazzolari and Carrera extended the hierarchical trigonometric Ritz formulation to deal with vibrations of multilayered shell [50]. Xie et al. [51] used the Haar Wavelet Method for computing natural vibrations of composite laminated cylindrical shells. Thinh and Nguyen [52] studied the dynamic response of a cross-ply laminated composite cylindrical shell with the dynamic stiffness method which is restricted to harmonic loadings that are necessarily located on the free edges of the shell. Distributed loads are not taken into account. This limitation does not allow dealing with cylindrical shells subjected, for example, to the action of external or internal fluids.

In this paper, the work is a direct continuation of a previous study [53] that proposed a procedure for taking into account distributed loads in the dynamic stiffness formulation in the case of axisymmetric shell structures made up with isotropic materials. In the present work, the method is extended to the case of cross-ply composite circular cylindrical shell as defined by Thinh and Nguyen.

The dynamic stiffness method [54–56] is based on exact relationships between harmonic loadings located on the boundaries of a structural element and the displacements of these boundaries. Recent formulations concern all kinds of structural elements such as functionally graded beams [57], composite plates [58–60] and stiffened [61] and composite shells [15]. To take into account distributed forces acting inside the geometrical domain, the main idea is to evaluate an equivalent loading located only on the edges. In this paper, the calculation of this equivalent loading is described and applied to thick cross-ply laminated composite cylindrical shells. Both rotatory inertia and shear deformation effects are taken into account. The accuracy of the proposed model is examined by comparing the solutions obtained using finite element models.

2. Elastodynamic problem

2.1. Geometry

The geometry of the cylindrical structural elements studied in this work is illustrated in Fig. 1.

The middle surface radius of the cylindrical shell is denoted R and its length L. The shell is made up of N orthotropic layers perfectly bound together. The kth layer is located between the coordinates ζ_k and ζ_{k+1} along the radial axis, therefore its thickness is given by: $h_k = \zeta_{k+1} - \zeta_k$. For general cross-ply laminated shells, the constitutive orthotropic material of the kth layer is oriented with an angle θ_k. This angle is equal to 0° or 90° about the cylinder axis. A local basis $(\mathbf{e_s}, \mathbf{e}_\theta, \mathbf{n})$ is defined on each point of the middle surface, see Fig. 1.

The position of each point of the shell is described in an orthogonal curvilinear system by three coordinates (s, θ, ζ) where s is the longitudinal coordinate of the point along the axis of the shell, θ is the angular circumferential coordinate and ζ is the coordinate along n about the midsurface. These coordinates are such that:

$$\mathbf{OM}(s,\theta,\zeta) = \mathbf{OP}(s,\theta) + \zeta\mathbf{n} \tag{1}$$

P being the orthogonal projection of M on the middle surface of the shell and $\mathbf{OP} = R\mathbf{n} + s\mathbf{e_s}$.

In this curvilinear coordinate system, the expression of the gradient of a vector field $\mathbf{V} = V_s(s,\theta,\zeta)\mathbf{e_s} + V_\theta(s,\theta,\zeta)\mathbf{e}_\theta + V_\zeta(s,\theta,\zeta)\mathbf{e}_\zeta$ is given by Eq. (2):

$$[\nabla V] = \begin{pmatrix} \frac{\partial V_s}{\partial s} & \frac{1}{R+\zeta}\frac{\partial V_s}{\partial \theta} & \frac{\partial V_s}{\partial \zeta} \\ \frac{\partial V_\theta}{\partial s} & \frac{1}{R+\zeta}\left(\frac{\partial V_\theta}{\partial \theta} + V_\zeta\right) & \frac{\partial V_\theta}{\partial \zeta} \\ \frac{\partial V_\zeta}{\partial s} & \frac{1}{R+\zeta}\left(\frac{\partial V_\zeta}{\partial \theta} - V_\theta\right) & \frac{\partial V_\zeta}{\partial \zeta} \end{pmatrix}_{(\mathbf{e_s},\mathbf{e_n},\mathbf{n})} \tag{2}$$

2.2. Formulation of cross-ply laminated circular cylindrical shells

2.2.1. Kinematic assumptions

According to the Mindlin–Reissner theory, the amplitude of the harmonic displacement field of point M is given by Eq. (3):

$$\begin{cases} u_s(s,\theta,\zeta) = U_s(s,\theta) + \zeta\phi_\theta(s,\theta) \\ u_\theta(s,\theta,\zeta) = U_\theta(s,\theta) + \zeta\phi_s(s,\theta) \\ u_\zeta(s,\theta,\zeta) = U_\zeta(x,\theta) \end{cases} \tag{3}$$

where u_s, u_θ and u_ζ are displacement components along $\mathbf{e_s}$, $\mathbf{e}_\theta$ and $\mathbf{n}$ respectively. U_s, U_θ and U_ζ are amplitude displacement components of point $P(s,\theta,0)$ on the middle surface of the shell. ϕ_s, ϕ_θ are the rotations of the middle surface about $\mathbf{e_s}$ and $\mathbf{e}_\theta$, respectively.

With the assumption of small displacements, the strain–displacement relationship $\overline{\overline{\epsilon}} = \frac{1}{2}\left(\nabla\mathbf{U} + \nabla^T\mathbf{U}\right)$ is obtained by considering Eqs. (2) and (3). We obtain:

$$\begin{cases} \epsilon_{ss} = \frac{\partial u_s}{\partial s} = \frac{\partial U_s}{\partial s} + \zeta\frac{\partial \phi_\theta}{\partial s} \\ \epsilon_{\theta\theta} = \frac{1}{R+\zeta}\left(\frac{\partial u_\theta}{\partial \theta} + u_\zeta\right) = \frac{1}{R+\zeta}\left(\frac{\partial U_\theta}{\partial \theta} + \zeta\frac{\partial \phi_s}{\partial \theta} + U_\zeta\right) \\ \epsilon_{\zeta\zeta} = \frac{\partial u_\zeta}{\partial \zeta} = 0 \\ \gamma_{s\theta} = 2\epsilon_{s\theta} = \frac{1}{R+\zeta}\frac{\partial u_s}{\partial \theta} + \frac{\partial u_\theta}{\partial s} = \frac{1}{R+\zeta}\left(\frac{\partial U_s}{\partial \theta} + \zeta\frac{\partial \phi_\theta}{\partial \theta}\right) + \frac{\partial U_\theta}{\partial s} + \zeta\frac{\partial \phi_s}{\partial s} \\ \gamma_{s\zeta} = 2\epsilon_{s\zeta} = \frac{\partial u_s}{\partial \zeta} + \frac{\partial u_\zeta}{\partial s} = \phi_\theta + \frac{\partial U_\zeta}{\partial s} \\ \gamma_{\theta\zeta} = 2\epsilon_{\theta\zeta} = \frac{1}{R+\zeta}\left(\frac{\partial u_\zeta}{\partial \theta} - u_\theta\right) + \frac{\partial u_\theta}{\partial \zeta} = \frac{1}{R+\zeta}\left(\frac{\partial U_\zeta}{\partial \theta} - U_\theta - \zeta\phi_s\right) + \phi_s \end{cases} \tag{4}$$

2.3. Lamina constitutive relations

The stress–strain relations are the constitutive equations of the orthotropic kth layer. These equations are given by [2] and, in the case of a cross-ply laminated shell, they are reduced to:

$$\begin{Bmatrix} \sigma_{ss} \\ \sigma_{\theta\theta} \\ \tau_{\theta\zeta} \\ \tau_{s\zeta} \\ \tau_{s\theta} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11}^{(k)} & \overline{Q}_{12}^{(k)} & 0 & 0 & 0 \\ \overline{Q}_{12}^{(k)} & \overline{Q}_{22}^{(k)} & 0 & 0 & 0 \\ 0 & 0 & \overline{Q}_{44}^{(k)} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55}^{(k)} & 0 \\ 0 & 0 & 0 & 0 & \overline{Q}_{66}^{(k)} \end{bmatrix} \begin{Bmatrix} \epsilon_{ss} \\ \epsilon_{\theta\theta} \\ \gamma_{\theta\zeta} \\ \gamma_{s\zeta} \\ \gamma_{s\theta} \end{Bmatrix} \tag{5}$$

where $\overline{Q}_{ij}^{(k)}$ are the transformed stiffnesses of the kth lamina as a function of the orientation θ of the orthotropic material direction. The expressions of these stiffnesses are given in Appendix A.

2.4. Shell behaviour equations

Relations between internal forces and section displacements of the shell are the behaviour equations. These equations are given by considering the internal forces per unit length, with the definition given by:

$$[N_{ss}, N_{\theta\theta}, N_{s\theta}, N_{\theta s}] = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\sigma_{ss}\left(1+\frac{\zeta}{R}\right), \sigma_{\theta\theta}, \tau_{s\theta}\left(1+\frac{\zeta}{R}\right), \tau_{s\theta}\right] d\zeta \tag{6}$$