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An efficient simulation method for the first excursion problem of linear structures subjected to stochastic wind loads

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ABSTRACT

In this paper the first excursion problem of linear structures subjected to stochastic wind loads is considered. The geometry of the corresponding failure domain is further explored, with the finding that rotational relationship exists for the quadratic elementary failure domains comprising the overall failure domain. Besides, procedures using the gradient projection method are developed for obtaining the design point of the quadratic elementary failure domain, which can be used to modify the previously developed simulation method. An illustrative example shows the accuracy and the enhanced efficiency of the modified simulation method, compared with its original version.

1. Introduction

In reliability analysis of linear structures subjected to stochastic wind loads, one important problem is to determine the probability that any one of N_y concerned output responses, $Y_i(t), i = 1, \dots, N_y$ exceeds in magnitude some prescribed threshold b_i within a given time duration T:

$$P_F = P(F) = P\left(\bigcup_{i=1}^{N_y} \{\exists t \in [0, T] : |Y_i(t)| > b_i\}\right) \tag{1}$$

This problem is commonly known as first excursion problem or first passage problem and has attracted much attention since the pioneering work by Rice [23,24].

Simulation methods offer a good alternative for addressing this problem. In simulation methods, the stochastic wind excitation is modeled with the help of a number of random variables $\{Z_1, \dots, Z_{N_z}\}$, which can be grouped in an N_z-dimensional random vector $\mathbf{Z}$. In this paper, we assume a deterministic linear structural model, and are concerned only with the uncertainties in the excitation loads. In the case where the uncertainties in excitation and structural parameters are both considered, some efforts for assessing first excursion probabilities can be found in [11,20,32]. In terms of the joint probability density function (PDF) $f(\mathbf{Z})$ of the random variables, the failure probability can be written as:

$$P(F) = \int_{\mathbb{R}^{N_z}} I_F(\mathbf{Z}) f(\mathbf{Z}) d\mathbf{Z} \tag{2}$$

where I_F is the indicator function that specifies the state of the structure for the excitation realized with a given $\mathbf{Z} : I_F = 1$ if the structure fails, in the sense that some of the structural responses exceed the prescribed thresholds, and $I_F = 0$ otherwise.

As implied by the above probability integral, the failure probability can be viewed as the expectation of $I_F(\mathbf{Z})$ with $\mathbf{Z}$ distributed as $f(\mathbf{Z})$. This perspective is the basis of Monte Carlo simulations (MCS) [21,22]. In MCS, the failure probability is estimated by the arithmetic average of the indicator function $I_F(\mathbf{Z})$ over the samples $\{\mathbf{Z}^{(r)} : r = 1, \dots, N_s\}$ generated according to $f(\mathbf{Z})$. The accuracy and efficiency of MCS does not depend on the geometry of the failure domain or the number of random variables involved in the problem. Instead, it depends only on the failure probability $P(F)$ and the number N_s of generated samples. The coefficient of variation (COV) δ of the MCS estimator for $P(F)$ is:

$$\delta = \sqrt{\frac{1 - P(F)}{P(F) N_s}} \approx \sqrt{\frac{1}{P(F) N_s}} \quad (P(F) \ll 1) \tag{3}$$

In engineering applications, the failure probability is expected to be small. To estimate the small failure probability with acceptable accuracy, the above equation implies that the required

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computational effort is prohibitively high. For example, if $P(F) = 10^{-4}$, one needs $N_s = 10^6$ samples (structural dynamic analyses) to achieve an accuracy level corresponding to 10% COV.

The first excursion problem is a high-dimensional reliability problem, since a large number of random variables are used for modeling the stochastic excitation. Several commonly used simulation algorithms, such as importance sampling, encounter serious difficulties when solving such high-dimensional reliability problems [3,13,27,28]. For the first excursion reliability problem of linear structures subjected to Gaussian stochastic loads, the failure domain can be described by a union of linear elementary failure domains with hyperplane boundaries [8]. In this case, Domain Decomposition Method (DDM) [12] was found to be an efficient simulation method. DDM is similar to Importance Sampling using Elementary Events (ISEE) [2] and a method presented in Yuen and Katafygiotis [33], although the approaches leading to each of these methods are different.

Herein, we consider the first excursion problem for linear structures subjected to stochastic wind loads. In this case, the failure domain is a union of quadratic elementary failure domains, rather than linear ones as assumed in the original formulation of DDM and ISEE. A simulation method using DDM and Line Sampling (LS) [15,19,26,27] has been previously developed by the authors for solving this reliability problem, and was shown to be highly efficient compared with MCS [14]. In that method, it is assumed that the elementary failure domains corresponding to a particular response and a particular crossing direction have the same probability and the overlapping degree among different elementary failure domains is time invariant. In this paper,the above assumption is confirmed through the exploration of the geometry of the failure domain. In addition, in that earlier paper for simplicity the linear coefficient vector in the limit state function is chosen to represent the important direction vector in LS. Here, the procedures using the gradient projection method are developed for determining the design point of the quadratic elementary failure domain in the high dimensional space, which provides a better choice for the important direction vector in the modified simulation method using DDM and LS. It is shown with an illustrative example involving an along-wind excited steel building that this modified simulation method can enhance the computational efficiency compared with its original version while retaining good accuracy.

2. Problem formulation

In this section, we give a brief formulation of the first excursion problem of linear structures subjected to stochastic wind loads, while for details one is referred to [14]. Consider a linear structure subjected to along-wind excitations, where the wind velocity field is discretized at N_u heights $h_j, j = 1, \ldots, N_u$, and N_y dynamic responses, $Y_i(t), i = 1, \ldots, N_y$, are of interest. The mean components $\overline{V}_j = \overline{V}(h_j)$ of the wind velocity processes $V_j(t) = V(h_j; t)$ at the specified heights $h_j, j = 1, \ldots, N_u$ are time invariant, and can be determined according to the assumed wind profile in the code. The fluctuating components of the wind velocity processes, $v_j(t) = v(h_j, t), j = 1, \ldots, N_u$, can be grouped in a stochastic vector process $\mathbf{v}(t) = [v_1(t), v_2(t), \ldots, v_{N_u}(t)]^T$, and simulated according to its one-sided cross-power spectral density matrix function $\mathbf{S}^0(\omega)$.

Using the spectral representation method [7,9,29,30] with random amplitudes, each component $v_j(t), j = 1, \ldots, N_u$ of the stochastic vector process $\mathbf{v}(t)$ can be simulated by the following superposition of harmonic waves:

$$v_j(t) = \sqrt{\Delta\omega}\sum_{l=1}^{N_\omega}\sum_{d=1}^{N_u}\left[H_{jd}(\omega_l)Z_{1ld}\cos(\omega_l t) + H_{jd}(\omega_l)Z_{2ld}\sin(\omega_l t)\right] \tag{4}$$

where the concerned interval $[0, \omega_u]$ from zero to the cutoff frequency ω_u is divided into N_ω equal segments, each having length $\Delta\omega = \omega_u/N_\omega$. The sequence of frequencies $\omega_l = (2l-1)\Delta\omega/2$, $l = 1, \ldots, N_\omega$ are the midpoints of the equal segments. $H_{jd}(\omega_l)$ is the element at j-th row and l-th column of the matrix $\mathbf{H}(\omega_l)$, which is the lower triangular matrix in the Cholesky's decomposition [10] of $\mathbf{S}^0(\omega_l)$ at frequency ω_l. Z_{1ld} and $Z_{2ld}, l = 1, \ldots, N_\omega, d = 1, \ldots, N_u$, are independent standard normal random variables, introduced to model the random amplitudes.

Here the wind velocity processes are assumed stationary, which is common practice for structural design in normal wind conditions. In extreme wind conditions such as hurricane and thunderstorm winds, non-stationarity has to be considered in wind velocity process models to reflect time varying statistics [1]. The simulation method in this paper is developed for the first excursion problem with stationary wind velocity processes, and the case considering non-stationarity is left for future research work.

With the simulated wind velocity process $V_j(t) = \overline{V}_j + v_j(t)$, the discretized wind excitation forces $U_j(t), j = 1, \ldots, N_u$ can be expressed as

$$U_j(t) = \frac{1}{2}\rho L_j \overline{V}_j^2 + \rho L_j \overline{V}_j v_j(t) + \frac{1}{2}\rho L_j v_j(t)^2 \tag{5}$$

where ρ is the air density and L_j is the area upon which the discretized force $U_j(t)$ is assumed to act. Correspondingly, N_y dynamic responses of interest $Y_i(t), i = 1, \ldots, N_y$ can be obtained by the following integral:

$$Y_i(t) = \sum_{j=1}^{N_u}\int_0^t q_{ij}(t,\tau)U_j(\tau)d\tau \tag{6}$$

where $q_{ij}(t,\tau) = q_{ij}(t-\tau)$ is the impulse response function for Y_i at time t due to a unit impulse excitation for U_j at time τ. The structural system considered herein is assumed to be time invariant and causal, and start with zero initial conditions. Substituting Eqs. (4) and (5) into Eq. (6), the response $Y_i(t)$ can be written as

$$Y_i(t) = Y_i^{(0)} + Y_i^{(1)}(t) + Y_i^{(2)}(t) = Y_i^{(0)} + \mathbf{a}^{(i)}(t)^T\mathbf{Z} + \mathbf{Z}^T\mathbf{B}^{(i)}(t)\mathbf{Z} \tag{7}$$

where $Y_i^{(0)}$ is the deterministic part of $Y_i(t)$, which corresponds to the mean wind velocity excitations and can be obtained by a single structural static analysis. The parts $Y_i^{(1)}(t) = \mathbf{a}^{(i)}(t)^T\mathbf{Z}$ and $Y_i^{(2)}(t) = \mathbf{Z}^T\mathbf{B}^{(i)}(t)\mathbf{Z}$ correspond respectively to the excitations due to the linear and the quadratic terms of the wind velocity fluctuation, and can be expressed as linear and quadratic functions of the standard normal random vector $\mathbf{Z}$ comprising all Z_{1ld} and Z_{2ld}.

Specifically, the part $Y_i^{(1)}(t), i = 1, \ldots, N_y$, is

$$Y_i^{(1)}(t) = \mathbf{a}^{(i)}(t)^T\mathbf{Z} = \sum_{l=1}^{N_\omega}\sum_{d=1}^{N_u}\left(a_{1ld}^{(i)}(t)Z_{1ld} + a_{2ld}^{(i)}(t)Z_{2ld}\right) \tag{8}$$

with the elements of the coefficient vector $\mathbf{a}^{(i)}(t)$ given by

$$a_{1ld}^{(i)}(t) = \sum_{j=1}^{N_u}\sqrt{\Delta\omega}\rho L_j\overline{V}_j H_{jd}(\omega_l)|Q_{ij}(\omega_l)|\cos\left(\omega_l t + \theta(Q_{ij}(\omega_l))\right) \tag{9}$$

and

$$a_{2ld}^{(i)}(t) = \sum_{j=1}^{N_u}\sqrt{\Delta\omega}\rho L_j\overline{V}_j H_{jd}(\omega_l)|Q_{ij}(\omega_l)|\sin\left(\omega_l t + \theta(Q_{ij}(\omega_l))\right) \tag{10}$$