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Optimal dividend strategies with time-inconsistent preferences

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ABSTRACT

This paper studies the optimal dividend strategies of an insurance company when the manager has time-inconsistent preferences. We consider the problem for a naive manager and a sophisticated manager, and analytically derive the optimal dividend strategies when claim sizes follow an exponential distribution. Our results show that the manager with time-inconsistent preferences tends to pay out dividends earlier than her time-consistent counterpart and that the sophisticated manager is more inclined to pay out dividends than the naive manager. Furthermore, we extend these results to the case with claim sizes following a mixed exponential distribution, and provide a numerical analysis to reveal the sensitivity of the optimal dividend strategies to changes in the premium, claims and surplus volatility.

1. Introduction

With the recent development of mathematical tools for financial engineering and actuarial science, there has been much research into optimal dividend strategies, especially, for insurance companies (see Avanzi, 2009; Albrecher and Thonhauser, 2009 for a review). These studies use the Cramér–Lundberg model or its diffusion approximation to describe the surplus of an insurance company. Their objective is to maximize the expected present value of future dividend pay-outs until the time of ruin, which is defined as the first time when the company's surplus becomes negative.[1]

The optimal dividend problem for the Cramér–Lundberg model is first solved by Gerber (1969) using the limit of an associated discrete-time problem and later by Schmidli (2008) using stochastic control. It is also solved using a viscosity solution by Azcue and Muler (2005), who consider the possibility of a general reinsurance strategy. All of these studies show that the optimal dividend strategy is a *band strategy*.[2] The optimality of the band strategy is also proven by Azcue and Muler (2012) for the case in which dividend rates are upper bounded. Although a band strategy is optimal for a general dividend

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[1] In line with the classical theory of Modigliani and Miller (1958), this approach can be used to determine a company's value. The optimal dividend problem has thus become an essential issue in corporate finance and actuarial mathematics (Holt, 2003).

[2] See Azcue and Muler (2005) and Schmidli (2008) for a rigorous formulation of the band strategy.

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problem in theory, its structure is too complicated in industry. Gerber (1969) shows that, in the case of exponentially distributed claim sizes, the optimal dividend strategy collapses to a so-called *barrier strategy*.[3] Barrier strategies are practical and thus have been widely studied by Loeffen (2008), Bai and Guo (2010), Kyprianou et al. (2012) and Hunting and Paulsen (2013), among others.[4]

In the discussed literature, future dividend payments are discounted exponentially at a constant discount rate. The company's manager thus has a constant rate of time preferences. These preferences are *time-consistent*, in that the manager's preferences regarding future dividend payments at an earlier date over a later date are the same (Grenadier and Wang, 2007). However, experimental study on time preferences demonstrates that the standard assumption of time-consistent preferences is unrealistic and that human beings' preferences for future rewards change with time (see Thaler, 1981; Ainslie, 1992; Loewenstein and Prelec, 1992). These preferences are called *time-inconsistent preferences*. In fact, there is substantial evidence showing that the discount rate is a decreasing function of time, which means that people are impatient about their choices in the short term but are patient about long-term alternatives.[5]

When an optimization problem has time-inconsistent preferences, the value function lacks the iterated-expected property and Bellman's optimality principle does not hold. Thus the optimization problem is called a time-inconsistent problem, and we cannot directly solve it using the dynamic programming approach (Björk and Murgoci, 2010). Strotz (1956) suggests three approaches to deal with the time-inconsistent problems: the first approach is to adopt some technologies (for example, by signing a contract) so that a manager's future behavior is irrevocable, the second approach is to ignore the conflict as a spendthrifty by assuming the decision-maker to be naive, and the third approach is to consider a strategy of consistent planning by assuming the decision-maker to be sophisticated. With the first two approaches, the optimal strategies are time-inconsistent since an optimal strategy determined at a particular moment is not necessarily optimal at a later moment.[6] However, the third approach imposes that the manager should take into account her future actions induced by her changing preferences. The third approach generates a time-consistent strategy and is generally implemented by taking the game theory point of view and considering the so-called subgame perfect Nash equilibrium strategies.[7] Strotz (1956) is the first to analyze the behavior of a decision-maker with time-inconsistent preferences, followed by Karp (2005) in the study of environmental regulation, Grenadier and Wang (2007) in the framework of real options, Ekeland et al. (2011) in the fishery resource management problem, and Harris and Laibson (2013) in a classical consumption problem.

In this paper, we revisit the optimal dividend problems with time-inconsistent preferences using the three approaches discussed above, to illustrate the effect of time-inconsistent preferences on an insurance company's dividend distribution. We assume that the company's surplus is described by the Cramér–Lundberg model with a diffusion component and that the dividends are paid out according to the barrier strategies. Following Grenadier and Wang (2007), Hsiaw (2013) and Harris and Laibson (2013), we model the time-inconsistent preferences of the company's manager using a continuous-time version of the *quasi-hyperbolic discount function*. Following the standard protocol for studying time-inconsistent behavior, we formally model the manager as a sequence of temporal selves, who have different interests and make decisions in a dynamic game. Thus the dividend problem can be seen as an intra-personal game between the successive selves.

We first consider the time-consistent problem as a benchmark, and based on it we further consider the optimization problems for cases in which the manager is either naive or sophisticated, depending on what the current self envisions about the preferences and behavior of her future selves. The current self of a naive manager pays out dividends without realizing that her future selves may have different preferences. As a result, she continuously modifies her dividend strategy, which is generally time-inconsistent. In contrast, the current self of a sophisticated manager correctly foresees that her future selves will have different preferences and will pay out dividends according to a time-consistent barrier strategy. We first transform both optimization problems into standard singular control problems and then derive the Hamilton–Jacobi–Bellman (HJB) equations using the dynamic programming approach. When the claim sizes are exponentially distributed,

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[3] A barrier strategy involves paying out all of the surplus exceeding $b \geq 0$ as dividends and doing nothing if the surplus is below b, where b is called the dividend barrier.

[4] Attempts have been made to use the diffusion approximation risk model to depict the surplus of an insurance company, such as by Jeanblanc-Picqué and Shiryaev (1995), Højgaard and Taksar (1999), Cai et al. (2006), Sotomayor and Cadenillas (2011) and Chen et al. (2013), among others. These results also show that a barrier strategy is optimal.

[5] For instance, people may prefer to get two oranges in 21 days rather than one orange in 20 days, but may also prefer to get one orange immediately than two oranges tomorrow, see Ekeland et al. (2012). This evidence is known as the *common difference effect* and can be explained by the dual system theory, see Shefrin and Thaler (1988) and Loewenstein and Prelec (1992).

[6] In a dynamic optimization problem, if the strategy π_{t_1} is optimal for the decision-maker at some time t_1 and there exists at least one time $t_2 > t_1$ such that the strategy π_{t_1} is not optimal for the decision-maker at time t_2, then it is called a *time-inconsistent strategy*. Otherwise, if for any $t_2 > t_1$, $\pi_{t_2}(t) \equiv \pi_{t_1}(t)$ for all $t \geq t_2$, the strategy is called a *time-consistent strategy*. If the strategy $\pi = \{\pi_t\}_{t \geq 0}$ is time-inconsistent, for $t \geq t_2$, the strategy π_{t_1} previously decided at time t_1 will not be implemented unless some commitment mechanism exists or the decision-maker is self-controlled (Hsiaw, 2013), otherwise, the decision-maker must look for a second-best strategy. There are resources other than time-inconsistent preferences that also lead to time-inconsistent problems, such as rank-dependent utilities and probability weighting, etc., see Hu et al. (2012), He and Zhou (2013), Zeng et al. (2013), and Björk et al. (2014). With the first approach, the decision-maker chooses a strategy that is optimal at the start and disregards whether that strategy is optimal at later times. With the second approach, the decision-maker chooses a strategy that is optimal on the first day, but will give up this strategy and choose a different one that is optimal on the second day. In the time-inconsistent preferences framework, both approaches generate time-inconsistent strategies.

[7] Under the game theory framework, the optimal strategy is derived as follows. At time t_1, the decision-maker considers that, given any time $t_2 > t_1$, starting from t_2 she will follow the strategy that is optimal at time t_2: $\pi_{t_2}(t) = \pi_{t_1}(t)$ for all $t \geq t_2$, where π_{t_1} and π_{t_2} are the corresponding optimal strategies at time t_1 and t_2, respectively. The derived optimal strategy is a time-consistent equilibrium strategy, see Björk and Murgoci (2010), Ekeland et al. (2012) and Bensoussan et al. (2014).