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Cross-hedging minimum return guarantees: Basis and liquidity risks

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ABSTRACT

We reveal pitfalls in the hedging of insurance contracts with a minimum return guarantee on the underlying investment, e.g. an external mutual fund. We analyze basis risk entailed by hedging the guarantee with a dynamic portfolio of proxy assets for the funds. We also take account of liquidity risk which arises since the insurer may need to advance funds for performing the hedge. Based on a least-squares Monte Carlo simulation, we study the economic implications of basis and liquidity risks. We demonstrate that both risks may be surprisingly high and show how the design of the contract *and* the hedging strategy may help to alleviate them.

1. Introduction

Complex life insurance products such as variable annuities (VAs) are well-known for the high demands they put on risk management. Frequently, their inherent risks were the source of considerable distress for insurers.[1] This is also reflected in one of the central goals of Solvency II: the market consistent valuation of embedded options within life insurance contracts. Such a risk-based regulatory framework aims at developing adequate hedging and reserving strategies to enforce policyholder protection. For this purpose, it is crucial to identify inherent risks and to assess hedging errors resulting from the inevitably imperfect hedging strategies. This is a demanding task already in simple setups, but even more so for “tailor-made” insurance contracts that have become popular in recent years.

On the one hand, contract designs suggested by the marketing and sales departments can entail hidden pitfalls for risk management. On the other hand, risk managers may suggest products which are easy to hedge but do not sufficiently address the prospective customer needs and expectations. Thus, an integrated approach to the design of such products is

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[1] Axa justified the retraction of “Axa Twinstar” with increased guarantee costs due to the financial market crisis. Since the 1980s there is a long list of defaulted life insurance companies in Europe, Japan and USA, see www.bankruptcydata.com.

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indispensable. This also requires an efficient and flexible numerical method for assessing and quantifying the risks and pitfalls of complex insurance products, hence enabling the monitoring of contract designs beforehand. In this paper, we study the benefits and implications of such an integrated risk management approach for guarantee contracts: we show that if guarantee contracts are not carefully designed they entail huge risks that cannot be controlled by standard hedging strategies.

Guarantee contracts, such as variable annuities (VAs), provide the policyholder upward participation in a risky investment while being protected from losses by a minimum return guarantee. From the insurer's perspective these guarantees represent embedded put options paid by the policyholder through guarantee charges. The options have a maturity and a complexity exceeding those of liquidly traded standard options by far. The embedded put option is written on an illiquid mutual funds, thus the insurer is exposed to *basis risk*. Basis risk turned out to be one of the key issues to trouble insurers during the recent financial market crisis.[2] The put option may be hedged, e.g., with liquid futures contracts on the EuroStoxx 50.[3] However, the actively managed equity fund underlying the guarantee contract is unlikely to contain exactly the same assets with the same weights as the EuroStoxx 50. Often, the insurer may not even have access to the current asset mix in the external fund. Thus, basis risk is a crucial issue for the external funds included in most guarantee contracts.

Under basis risk, hedging can only reduce the risk of a position to some extent and, accordingly, perfect replication cannot serve as a guideline for the hedging approach. In this paper, we focus on hedging strategies that minimize the residual variance of the embedded put option. Our approach can be interpreted as a continuous-time forward-looking counterpart to the "fund mapping" hedging strategies applied in practice. In fund mapping, the hedge ratio is calculated through a linear regression based on historical data. To simplify the exposition, our numerical example is based on a Black–Scholes framework, but it can be easily extended to more sophisticated models. The paper contributes to the existing literature on basis risk in the following ways: First, we stress that the basis risk in typical guarantee contracts tends to be substantial, for example, a correlation between underlying and hedging instrument of 88% implies that hedging reduces the position's standard deviation only by about 50%. In particular, we confirm numerically a simple rule of thumb for the amount of basis risk, which is well-known and exact in the case where the hedging instrument is a martingale. Lastly, we provide tight bounds on the hedging error's standard deviation which greatly simplify the design and the evaluation of numerical simulation methods. In particular, the overall approach easily extends different settings and contract designs.

Typically, the guarantee is financed through the regular deduction of a charge, calculated, e.g., as a proportion of the current account value or as a fixed monetary amount. Therefore, the capital required for hedging the option only becomes available slowly over time. The insurer may need to credit-finance part of the hedging portfolio and hence faces *liquidity risk*. We show that the patterns of liquidity risk we observe are very sensitive to the mechanism used for deducting the charges.

Our main economic insights concerning the interplay of basis and liquidity risks are the following. Our analysis shows that changes in contract design which are innocent in a complete market framework turn out to have significant implications for risk and its management. For instance, *proportional* (performance-based) guarantee charges amplify liquidity and basis risks compared to *fixed* guarantee charges. We show that increasing guarantee charges hardly alleviates basis and liquidity risks and hence is not a suitable tool neither for marketing nor for risk management. Moreover, we argue that working with misspecified model parameters has small impact on the hedging performance, but a strong impact on estimated basis and liquidity risks.

1.1. Related literature

Our paper is related to different strands of the literature. In the literature on life insurance contracts with guarantees it was recognized early on, e.g. Brennan and Schwartz (1976), that the embedded financial derivatives must be evaluated according to "no arbitrage" arguments. In a complete market setup every financial claim can be replicated by a self-financing strategy in the underlying assets. Here, the price for the guarantee matches the price for the embedded put option component. The pricing of unit-linked products is analyzed to a great extent in the literature, see for an overview Schneider (2012). Periodic premia introduce an Asian option feature which impedes closed-form solutions. The pricing of Asian options is extensively discussed in the literature. See Curran (1994), Rogers and Shi (1995), Nielsen and Sandmann (2002) or Chen et al. (2008) and the references therein. An application in the context of guarantee contracts with periodic premia can be found in Schrager and Pelsser (2004). Mahayni and Schoenmakers (2011) take into account additional tailor-made features (fund switching rights) and show that even the pricing becomes quite demanding.

The risk management of variable annuities is addressed in Coleman et al. (2006, 2007). Here the focus is also on risk-minimizing hedging strategies under stochastic volatility and jumps. However, these works do not address basis risk and, consequently, do not provide tools for analyzing the relevant risk measures numerically.

Realistic models often preclude closed-form solutions and thus efficient algorithms are essential for risk management. We approximate variance minimal hedging strategies using a least-squares approach put forward in a more abstract setting by Ankirchner et al. (2012b). We remove the assumption that the hedging instrument is a martingale which is crucial in

[2] For the importance of basis risk during the financial crisis, see "Managing Fund Mapping, Hedge Ratios and Basis Risk in Variable Annuities", Zhang (2012), Publication from the Society of Actuaries, www.soa.org.

[3] Futures contracts on indices are typically very liquid. See "Pricing Variable Annuity", www.towerswatson.com.