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Best response dynamics with level-n expectations in two-stage games

Herbert Dawid [a], Dennis Heitmann [b]

[a] Department of Economics and Business Administration, Center for Mathematical Economics, Bielefeld University, P.O. Box 100131, 33501 Bielefeld, Germany

[b] Department of Economics and Business Administration, Bielefeld University, P.O. Box 100131, 33501 Bielefeld, Germany

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ABSTRACT

This paper analyzes behavior in repeatedly played two-stage games, where players choose actions in both stages according to best replies using level-n expectations about the opponent's actions in both stages. Level-n expectations are recursively defined in a way that a player holding level-n expectations correctly predicts the action of an opponent holding level-$(n-1)$ expectations. A general conceptual framework to study such dynamics for two-stage games is developed and it is shown that, contrary to results for single-stage games, the fixed points of the dynamics depend on the level of the expectations. In particular, for level-0 expectation, fixed points correspond to a Nash equilibrium of a simultaneous move version of the game, whereas (under certain conditions) fixed points converge towards the subgame perfect equilibrium of the two-stage game if the level of expectations goes to infinity. The approach is illustrated using a two-stage duopoly game, where firms in the first stage invest in activities reducing their marginal costs and in the second stage engage in Cournot competition. Conditions for local stability of the fixed points are derived for different levels of expectations and it is shown that level-2 expectations are sufficient to move the fixed-point of the dynamics to a close neighborhood of the subgame perfect equilibrium. Furthermore, it is demonstrated that although firms benefit from unilateral increases in the level of expectations, an increase of n by all firms reduces all profits.

1. Introduction

A large part of the theoretical and empirical analyses in many areas of economics relies on the assumption that the behavior of the involved decision makers is determined by some sort of equilibrium. In contexts, like Industrial Organization, where multi-stage games are frequently used to describe strategic interaction with sequential decisions, a standard equilibrium concept considered in the literature is that of subgame perfect equilibrium as a refinement of Nash equilibrium. The notion of (Nash) equilibrium rests on two basic assumptions. First, the assumption that each player is able to determine her payoff-maximizing strategy given the strategies of all other players and, second, that in equilibrium the expectations of each player concerning the strategies of all players coincide with the strategies actually used by these players. Both assumptions have been extensively discussed in the literature and considerable amount of research has analyzed the question under which circumstances players, who ex-ante cannot determine their best responses and/or do not have correct expectations about the strategies followed by the other players, can over time coordinate in a way such that

E-mail addresses: hdawid@wiwi.uni-bielefeld.de (H. Dawid), dheitmann@wiwi.uni-bielefeld.de (D. Heitmann).

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eventually a Nash equilibrium is played. These studies assume that the considered game is played repeatedly and that players adjust their strategies over time according to some given process.

The earliest stream of the literature dealing with the dynamic stability of Nash equilibria is work on best response dynamics, where it is assumed that each player is able to determine her best response correspondence, but ex-ante expectations about the other players strategy are in general not correct and are updated over time (see. e.g. Brown, 1951). A rich literature has studied the dynamic properties of best response dynamics under naive and adaptive expectations in the framework of oligopolistic market interaction (see the survey by Kopel, 2009) identifying conditions on the market environment and the expectation formation process under which Cournot, respectively Bertrand, equilibria are (locally asymptotically) stable fixed points of the corresponding best response dynamics. This entire literature on best response dynamics in oligopolies focuses, however, on one-stage games, which is quite restrictive since many important issues in oligopolistic competition, like capacity choice, location choice or cost-reducing activities of firms, are typically analyzed using multi-stage games.

The agenda of this paper is to relax this restriction to one-stage games and to extend the analysis of best response dynamics to a scenario where in each period firms interact in two sequential stages. Furthermore, we relax the assumptions of naive expectations of individuals, which is typically made in this literature, to a class of expectation formation functions denoted as level-n expectations, which include naive expectations as a special case.

In particular, we consider a standard two-stage model of duopolistic competition with differentiated products introduced in Qiu (1997), where in the first stage firms make cost-reducing investments and, in the second stage, they sell the product in the market. We consider a scenario where the demand function and the cost structure of both firms are common knowledge, but firms face strategic uncertainty in a sense that they do not know their opponent's actions in the current stage when making their decision. Hence, they build expectations about these actions and then choose their best response given these expectations. First stage actions are observable between the stages, which means that expectations about the opponents second stage action might be influenced by the first stage action of this player. Given our assumption that the payoff functions of both players are common knowledge, both players are in a position to determine not only the own best response function but also that of the opponent in both stages.

Consider a firm which expects that its competitor uses naive expectations about its own actions. Anticipating that this competitor will play the best response based on these naive expectations the firm is able to predict the action of its competitor for any realization of the previous period actions (since the previous period actions are common knowledge). Denoting naive expectations as level-0 expectations, we say that a firm following this kind of rationality has level-1 expectations. If the competitor indeed has naive (i.e. level-0) expectations, then these level-1 expectations are correct. Such a firm would then choose its actions as a best response to its level-1 expectations of the opponent's action. Any firm which would correctly predict the actions of a firm with level-1 expectations is said to have level-2 expectations and extending this reasoning we define level-n expectations for an arbitrary integer n.

The paper puts forward four main findings. First, it is shown for a general class of two-stage games, which includes the duopoly model with cost-reducing investments as well as many other standard models in Industrial Organization, that, if the expectation formation follows the standard assumption of level-0 (i.e. naive) expectations, then the fixed point of the best response dynamics corresponds to a Nash equilibrium of a version of the game, where first-stage actions are not revealed before the second stage. Such equilibria are in general not subgame perfect. Put differently, strategic effects of first stage actions on second stage outcomes have no role under level-0 expectations. Second, it is shown again for the general class of games that such strategic effects do play a role for the determination of the fixed point of the dynamics if firms have expectations of higher level. As the level of expectations tends to infinity the fixed points of the dynamics converge to subgame perfect equilibrium outcomes. In the framework of the duopoly with cost-reducing activities it is demonstrated that already for level-2 expectations the unique fixed point of the dynamics is very close to the unique subgame perfect equilibrium. Third, increasing the level of expectations tends to destabilize the fixed points of the dynamics in the sense that there are parameter constellations where the fixed points are stable for an expectation level of zero but unstable if the level of expectation formation is large. Fourth, if the level of expectation formation goes up in the duopoly with cost-reducing activities this induces an increase in the long run consumer surplus however a decrease in the long run profits of the firms. Intuitively, more sophisticated expectation formation implies that cost-reducing activities in stage one have a stronger impact on the opponent's quantities in stage two. Therefore, incentives to reduce marginal costs go up and costs as well as prices decrease. If a firm however unilaterally increases its level of expectation formation this has positive effects on its long run profits.

These results are helpful for disentangling two implications of the revelation of first-stage actions before the second stage. The first implication of the revelation, which is that a firm can observe the opponent's first-stage action before choosing the own second-stage action, turns out to be irrelevant for the fixed point (but not for the transient dynamics) of the best reply dynamics if expectations in stage two are naive. Although firms for any level of expectations fully take the observed opponent's first-stage action into account when determining their best reply, for expectation level smaller than two the fixed point coincides with the one of the corresponding best reply dynamics, where the first-stage actions are not revealed in the second stage. Crucial for the location of the fixed point of the dynamics is the second implication of the revelation of first-stage actions, namely that firms expect that their own first-stage action influences the opponent's second-stage choice via the opponent's second-stage expectation formation. This effect becomes stronger the higher the level of the expectations and for large expectation levels approaches the full-fledged inter-stage strategic effect present in the subgame perfect equilibrium.