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Optimal timing for annuitization, based on jump diffusion fund and stochastic mortality

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ABSTRACT

Optimal timing for annuitization is developed along three approaches. Firstly, the mutual fund in which the individual invests before annuitization is modeled by a jump diffusion process. Secondly, instead of maximizing an economic utility, the stopping time is used to maximize the market value of future cash-flows. Thirdly, a solution is proposed in terms of Expected Present Value operators: this shows that the non-annuitization (or continuation) region is either delimited by a lower or upper boundary, in the domain time-assets return. The necessary conditions are given under which these mutually exclusive boundaries exist. Further, a method is proposed to compute the probability of annuitization. Finally, a case study is presented where the mutual fund is fitted to the S&P500 and mortality is modeled by a Gompertz Makeham law with several real scenarios being discussed.

1. Introduction

Buying a fixed-payout life annuity is an efficient solution to preserve standards of living during retirement and it also protects individuals against poverty in old age. The main drawbacks of this type of insurance are its irreversibility and the fact that payments are contingent on the recipient's survival. On the other hand, insurance companies or banks distribute financial products based on mutual funds, designed for people willing to take more risk with their money in exchange for a larger growth potential of their investments. In this context, the literature provides a great deal of evidence that pre-retirement people should invest in such schemes rather than in life insurance products. The question then arises whether and when to switch from such a financial investment to a life annuity.

Numerous papers have covered the various aspects of the annuitization problem since the well-known paper of Yaari (1965), which showed that individuals with no bequest motive should annuitize all their wealth at retirement. By using a shortfall probability approach, Milevsky (1998) considers by the setting up of a Brownian motion fund and using CIR interest rates, the probability of successful deferral, i.e. to defer annuitization as long as investment returns guarantee an income at least equal to that provided by the annuity. Milevsky et al. (2006) derive the optimal investment and annuitization strategies for a retiree whose objective is to minimize the probability of lifetime ruin. Hainaut and Devolder (2006) present a numerical study on the optimal allocation between annuities and financial assets when considering a utility maximization problem. Stabile (2006) examined the optimal annuitization time for a retired individual who is subject to the constant force

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of mortality in an all-or-nothing framework (i.e. the individual invests all his wealth to buy the annuity) with different utility functions for consumption before and after annuitization. Milevsky and Young (2007) examined optimal annuitization strategies for time-dependent mortality functions based on maximizing the returns from the investment in the case of the all-or-nothing context compared to the case when the individual can annuitize fractions of his wealth at any time. Emms and Haberman (2008) discuss both the optimal annuitization timing and the income draw-down scheme by minimizing a loss function and by using the Gompertz mortality function and a fund based on the Brownian motion. Purcal and Piggott (2008) explain the low annuity demand by the relative importance of pre-existing annuitization and by considering utility maximization, a geometric Brownian motion modeling the fund and mortality tables. Horneff et al. (2008) study, using a discrete time model, the optimal gradual annuitization for a retired individual applying Epstein–Zin preferences and quantifying the costs of switching to annuities. Gerrard et al. (2012) take the problem of maximizing the value of the investment to analyze (using a Brownian model and with constant force of mortality) the optimal time of annuitization for a retired individual managing his own investment and consumption strategy. Di Giacinto and Vigna (2012) consider a member of a defined contribution pension fund who has the option of taking programmed withdrawals at retirement. They then explore the sub-optimal cost of immediate annuitization, when minimizing a quadratic cost criterion in a Brownian motion setting and with a constant force of mortality. Huang et al. (2013) are also interested in the problem of optimal timing of annuitization, and especially in the optimal initiation of a Guaranteed Lifetime Withdrawal Benefit (GLWB) in a Variable Annuity. They focus on the problem from the perspective of the policyholder (i.e. when to begin withdrawals from the GLWB) and they adopt a No Arbitrage perspective (i.e. they assume that the individual is trying to maximize the cost of the guarantee to the insurance company offering the GLWB). Huang et al. (2013) provide a detailed and relevant overview of the literature concerning Variable Annuities and their guarantees.

This paper looks at the optimal timing to switch from a financial investment to a life annuity. It differs from previous publications in several ways. Firstly, the financial asset into which the individual invests (before transferring to annuitization) is modeled by a jump diffusion process instead of a geometric Brownian motion. Numerical applications, by which the return from this asset is fitted to the S&P500 index, reveal that the presence of jumps modifies significantly the point of switching, when compared with the prediction from a Brownian model. Secondly, instead of maximizing an economic utility, the stopping time maximizes the market value of future cash-flows.

When the discount rate is equal to the risk free rate, the objective is the market value or price of future expected discounted payouts. Huang et al. (2013) use a similar criterion for GLWB annuities and interpret it as the cost to the insurance company that provides this service. The investor acts to maximize this cost. In this case and as detailed in the body of the paper, this cost is split into an immediate lifetime payout annuity and an option to defer this annuity. By analogous to a classical American option, the annuitization should only be exercised once the value from waiting is zero, at a point in time when the asset value or return crosses a boundary. Stanton (2000) used a similar approach to estimate long-lived put option, embedded in 401(k) pension plans.

Since this problem has similarities with American option pricing, this paper proposes a semi-closed form solution in terms of Expected Present Value (EPV) operators, such as defined by Boyarchenko and Levendorskii (2007). However, for American options pricing, we know beforehand if the boundary delimiting the exercise region is an upper (call) or a lower (put) barrier, in the domain time-accrued return. However, in the current approach, this aspect would not be known at the beginning. On one hand, a basic reasoning suggests that one should consider switching to annuity if the financial asset performs poorly due to the fear of subsequent erosion of wealth. In this respect the non-annuitization (or continuation) region should be delimited by a lower boundary, in the space time versus realized returns. On the other hand, another reasoning leads to consider changing to annuitization when the realized financial return is high enough to receive a reasonable annuity. In this case, the continuation region should be delimited by an upper boundary. The originality of the current study is to present necessary conditions under which these mutually exclusive boundaries exist and a method to compute them.

This reasoning is sustained by empirical observations. Stanton (2000) mentions that in September and October, 1998, more than three times as many pilots of American Airlines retired as during an average month. According to the Wall Street Journal, this surge in retirements was occurring because pilots retiring at this date can take away retirement distributions based on July's high stock-market prices. Similar accelerated retirements occurred after the stock market crash of 1987. On Monday November 2, 1987, over 600 Lockheed Corp. employees had submitted early retirement papers the previous Friday, October 30 (approximately three times the usual monthly figure). Stanton (2000) determines in a Brownian framework that the investor optimally exercises the option to time their retirement or rollovers to another plan if the asset value crosses a boundary.

A third contribution is the assumption of a time dependent current force of mortality, which is contrary to many existing papers (e.g. Stabile, 2006, Gerrard et al., 2012). Finally, this paper proposes a method to estimate numerically the probability of annuitization. Of special note is that the solution based on expected value operators can be extended to constant and time dependent consumption/contribution rates, or to planned lump sum payments before annuitization. However, the proposed method does not allow one to dynamically manage the consumption.

Section 2 of this paper presents the dynamics of the financial asset into which the individual invests his savings, before annuitization. Section 3 discusses the current assumptions related to the mortality process. Section 4 introduces the maximization problem and in particular the objective function. Section 5 reviews the basic working of the Wiener–Hopf factorization that is used in Section 6 to locate the optimal annuitization time. Section 7 presents the Laplace transform of