Journal of Economic Dynamics & Control 38 (2014) 65–71

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Dynamic asset allocation when bequests are luxury goods

Jie Ding, Geoffrey Kingston*, Sachi Purcal

Department of Economics, Macquarie University, Sydney NSW 2109, Australia.

ARTICLE INFO

Article history:
Received 14 December 2012
Received in revised form
11 August 2013
Accepted 4 September 2013
Available online 13 November 2013

JEL classification:
D14
G11
G23

Keywords:
Bequests
Luxury goods
Dynamic asset allocation
Merton portfolio problem
European put option
Retirement risk zone

ABSTRACT

Luxury bequests impart systematic effects of age to an investor's optimal allocation: the expected percentage allocation to equities rises throughout retirement. When bequests are luxuries the marginal utility of bequests declines more slowly than the marginal utility of consumption. This is essentially lower risk aversion. As a retiree approaches death, her expected remaining lifetime utility is increasingly composed of bequest utility, and thus generates progressively lower risk aversion. A retiree responds by increasingly favoring the higher-return risky asset. Compared to standard preferences, luxury bequests elevate a retiree's average exposure to the risky asset, but the difference is small in early retirement.

1. Introduction

This paper offers the first analysis of the implications for dynamic asset allocation of bequests that are luxury goods.[1] Luxury bequests impart systematic effects of age to an investor's optimal allocation. In particular, a retiree's percentage allocation to equities rises with age.[2] By contrast, standard analysis highlights the case of a constant percentage exposure to equity. Sharper still is the contrast between the main result of this paper and a popular rule of thumb that says an investor's percentage allocation to equity should be set at 100 minus her age, even in retirement. Luxury bequests elevate a retiree's average exposure to risky assets, compared to the standard case where utility from bequests is treated as having the same

* Corresponding author. Tel.: +61 2 9850 7063; fax: +61 2 9850 6069.
E-mail address: geoff.kingston@mq.edu.au (G. Kingston).

[1] Atkinson (1971) and Davies (1982) invoke luxury bequests to explain persistent wealth inequality across generations in the face of regression to the mean in earnings. Menchik (1980) estimates that the elasticity of bequests with respect to lifetime resources is 2.5. Carroll (2002) invokes luxury bequests to explain the high average allocation to equity-type assets in the portfolios of the rich. Dynan et al. (2002) and Lockwood (2011, 2012) point out that luxury bequests and precautionary saving together help explain the facts of low voluntary annuitization and low take-up of long-term care insurance, especially by the upper half of the income distribution. They note that low drawdowns of financial assets by many retirees are consistent with this two-part explanation. Wachter and Yogo (2010) find that the share of risky assets in portfolios tends to rise with the investor's wealth, noting that this is evidence against homothetic preferences such as constant relative risk aversion. De Nardi et al. (2010) and Lockwood (2012) find evidence for luxury bequests based on the Method of Moments.

[2] Merton (1969, Footnote 5) says that if the bequest function is not of power form then "systematic effects of age will appear in the optimal decision-making." Our contribution verifies this observation and builds on Merton's analysis by characterizing the systematic effects of age when bequests are luxury goods.

0165-1889/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jedc.2013.11.004

power form as utility from non-bequest consumption. However, a numeric exercise suggests that the difference is small in early retirement, consistent with the notion that a retiree should have anupward-sloping age-equity profile.

Intuition for an expected high average exposure to investment risk late in life can be conveyed via a diagram for the stylized case when bequests are pure luxury goods.[3] See Fig. 1.

The horizontal axis shows wealth taken into retirement, before and after a negative wealth shock. The left-hand vertical axis shows the marginal utility of consumption in retirement. The right-hand vertical axis shows the marginal utility of the planned bequest, portrayed in the figure as a pure luxury good in the sense that the marginal utility of the planned bequest is constant, corresponding to a perfectly elastic demand for bequests with respect to wealth. Following a negative wealth shock, the planned bequest drops by an equal amount. This lowers the investor's welfare; the shaded area shows the welfare reduction. But consumption in retirement stays the same.

Fig. 1 also helps with intuition in the case of dynamic asset allocation. When bequests are a luxury good the marginal utility of bequests declines more slowly than the marginal utility of consumption. But this is essentially lower risk aversion. As a retiree approaches death, her expected remaining lifetime utility is increasingly composed of bequest utility, and thus generates progressively lower risk aversion. A retiree responds by increasingly favoring the higher-return risky asset.[4]

2. Model

We embed a bequest function, studied by Carroll (2002), De Nardi et al. (2010) and Lockwood (2011, 2012) into the portfolio model introduced by Merton (1969, 1971) and amended by Cox and Huang (1989). The investor makes contingent plans for a bequest $b(T)$, consumption rates $c(t)$, and proportionate investments $x(t)$ in risky assets, that maximize expected utility:

$$E\left[\int_0^T e^{-\rho t}\frac{(c(t)-h)^{1-\delta}}{1-\delta}dt+e^{-\rho T}\theta^{\delta}\frac{(\theta a+b(T))^{1-\delta}}{1-\delta}\right], \tag{1}$$

subject to a budget constraint:

$$dw(t)=[(x(t)(\alpha-r)+r)w(t)-c(t)]\,dt+x(t)w(t)\sigma\,dz(t), \tag{2}$$

and initial condition:

$$w(0)>\frac{h(1-e^{-rT})}{r}, \tag{3}$$

where the notation is E expectations operator, T age at death (assumed known), ρ rate of time preference, h nonnegative utility parameter with the interpretation of 'subsistence' or 'protected' or 'habitual' consumption,[5] δ positive utility curvature parameter, $\theta\equiv\phi/(1-\phi)$ transformation of a utility parameter $\phi\in(0,1)$ that has the interpretation of "the marginal propensity to bequeath in a one-period problem of allocating wealth between consumption and an immediate bequest" (Lockwood, 2012, p. 6),[6] a nonnegative bequest utility parameter, w wealth, α instantaneous expected return to risky assets, r return to safe assets, σ volatility of risky assets, and dz Wiener increment.

The bequest function in Eq. (1) can be related to Fig. 1. The first derivative of the bequest-utility function with respect to the amount of bequest b is

$$\theta^{\delta}(\theta a+b)^{-\delta}\geq 0, \tag{4}$$

[3] An optimum problem that is consistent with Fig. 1 is given by

$$\max_{b,c}-\frac{(\bar{c}-c)^2}{2}+\theta b$$

subject to

$$b+c=w$$

and

$$b,c,w>0.$$

The notation is b bequest, c consumption, w wealth taken into retirement, $\bar{c}$ satiety level of consumption and θ marginal utility of bequests (assumed constant).

[4] We thank a referee for suggesting this intuition for our main result.

[5] Wachter and Yogo (2010) introduce nonhomothetic preferences by distinguishing between necessities and luxuries in non-bequest consumption, noting but not invoking luxury bequests. We follow Merton (1971) in distinguishing between 'protected' and 'unprotected' consumption. This also generates a high share of risky assets in the portfolios of the rich, but simplifies analysis as it avoids the complication of a relative price between pure necessities and other goods in non-bequest consumption. 'Protected' consumption is a pure necessity as its elasticity of demand with respect to wealth is zero. This combination of protected consumption and a luxury good can impart a 'barbell' character to optimal asset allocation, reminiscent of portfolio-insurance strategies (see e.g. Black and Perold, 1992).

[6] See also Footnote 3 above.