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Computing equilibria in dynamic models with occasionally binding constraints

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ABSTRACT

We propose a method to compute equilibria in dynamic models with several continuous state variables and occasionally binding constraints. These constraints induce non-differentiabilities in policy functions. We develop an interpolation technique that addresses this problem directly: It locates the non-differentiabilities and adds interpolation nodes there. To handle this flexible grid, it uses Delaunay interpolation, a simplicial interpolation technique. Hence, we call this method Adaptive Simplicial Interpolation (ASI). We embed ASI into a time iteration algorithm to compute recursive equilibria in an infinite horizon endowment economy where heterogeneous agents trade in a bond and a stock subject to various trading constraints. We show that this method computes equilibria accurately and outperforms other grid schemes by far.

1. Introduction

In many applications of dynamic stochastic (general) equilibrium models, it is a natural modeling choice to include constraints that are occasionally binding. Examples are models with borrowing constraints, limited commitment, a zero bound on the nominal interest rate, or irreversible investments. These constraints induce non-differentiabilities in the policy functions, which make it challenging to compute equilibria. In particular, standard interpolation techniques using non-adaptive grids perform poorly both in terms of accuracy and shape of the computed policy function (see, e.g. Judd et al., 2003, pp. 270–1). This paper proposes a method that overcomes these problems, even for models with several continuous state variables. We call this method Adaptive Simplicial Interpolation (ASI). Its working principle is to locate the non-differentiabilities that are induced by occasionally binding constraints, and to put additional interpolation nodes there.

We present our algorithm in the setting of a dynamic endowment economy where three or four (types of) agents face aggregate and idiosyncratic risk. To explain the main features of ASI we first compute equilibria in a simple two period version where agents trade in a bond subject to an ad hoc borrowing constraint. Second, we embed ASI into a time iteration

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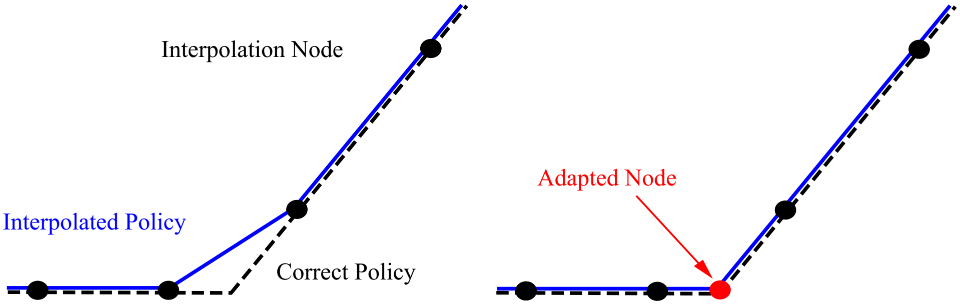

Fig. 1. Non-adaptive (lhs) and adaptive (rhs) linear interpolation in 1D.

algorithm to solve an infinite horizon version of the model. Finally, we add a Lucas tree-type stock, which is subject to a short sale constraint, and we replace the ad hoc borrowing constraint by a collateral constraint. Consequently, short positions in the bond need to be collateralized by stock holdings, while the stock may not be shorted.

Compared to earlier papers using a similar setup, such as Heaton and Lucas (1996), den Haan (2001) or Kubler and Schmedders (2003), the models we consider differ in two respects, which both make it harder to compute equilibria: First, we solve models with more agents, which result in a continuous state space of higher dimension. As the kinks[1] naturally form hypersurfaces in the state space, they are of higher dimension as well. Second, in our extension, the trading constraints that agents face depend on tomorrow's equilibrium price of the stock, which is endogenously determined. Consequently, it is much harder to locate the kink and ad hoc methods fail.

Fig. 1 illustrates the working principle of ASI. The dashed line displays a simple one-dimensional policy function with a kink. Suppose this function is approximated by linear interpolation between equidistant gridpoints. The resulting interpolated policy is displayed as a solid line on the left-hand side of Fig. 1. Clearly, the approximation error is comparatively large around the kink, and this is just because there is no interpolation node near the kink. If we knew the location of the kink and put a node there, then the approximation would be much better, as the right-hand side of Fig. 1 shows. This is the motivation for ASI, which directly addresses the problem of kinks in policy functions by placing additional gridpoints, called *adapted points*, at these non-differentiabilities. In higher dimensional state spaces and with complex constraints, this approach is not as simple as Fig. 1 suggests. Hence, we need a flexible interpolation technique and a systematic adaptation procedure.

To be able to place gridpoints wherever needed, we use *Delaunay interpolation*, which consists of two steps. First, the convex hull of the set of gridpoints is covered with simplices, which results in a so-called *tessellation*. Then we linearly interpolate locally on each simplex.[2]

We adapt the grid as follows: First, we solve the system of equilibrium conditions on an initial grid. Second, we use these solutions to determine which edges of the tessellation cross kinks. Third, on each of these edges, we solve a modified system of equilibrium conditions to determine the point of intersection with the kink. Finally, we place a new grid point there. Using this procedure with state spaces of more than one dimension, we get several adapted gridpoints for each kink. Delaunay tessellation connects these points by edges, such that the kinks are matched very accurately.

To solve the above described infinite horizon models, we embed adaptive simplicial interpolation in a standard time iteration algorithm (see, e.g. Judd, 1998). To assess the accuracy of the computed equilibria, we follow Judd (1992) in calculating relative errors in Euler equations, subsequently called *Euler errors*. Concerning the measured Euler errors, we find that our method accurately computes equilibria for the two economies considered, both for reasonable and extreme calibrations of our model. Furthermore, we assess the relative performance of the adaptive grid scheme by comparing it to a standard equidistant grid scheme using the same interpolation technique. We find that the adaptive grid scheme dominates by far: One needs to increase the number of equidistant gridpoints, and thereby computation time, by more than two orders of magnitude in order to reach the high accuracy of the adaptive grid scheme. Finally, we demonstrate that ad hoc update procedures that place additional points near the kinks are much less efficient than ASI.

In the literature, many algorithms have been applied to dynamic models with occasionally binding constraints. However, none of the existing algorithms address the problems of non-differentiabilities directly. Christiano and Fisher (2000) compare how several algorithms compute equilibria in a one sector growth model with irreversible investment, which has only one continuous state variable. None of the applied algorithms use an adaptive grid scheme. A grid structure which is not adaptive, but endogenous, is proposed by Carrol (2006) and extended by Barillas and Fernández-Villaverde (2007), Hintermaier and Koeniger (2010), and Ludwig and Schön (2013). This so-called endogenous grid method defines a grid on tomorrow's variables, resulting in an endogenous grid on today's variables. Its major advantage is that it avoids the root-finding step. Yet, as it exploits the specific mapping from next period's variables to today's variables, the applicability as

[1] In our terminology, a *kink* associated with a certain constraint is the set of points at which the policy function fails to be differentiable because the constraint is *just* binding, i.e. the constraint is binding *and* the associated multiplier is zero.

[2] Linear simplicial interpolation is only C^0 at the boundaries. For our purposes, this is desirable, because it provides a better fit at the kinks.