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Escape dynamics: A continuous-time approximation

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ABSTRACT

We extend a continuous-time approximation approach to the analysis of escape dynamics in economic models with constant gain adaptive learning. This approach is based on the application of the results of continuous-time version of large deviations theory to the linear diffusion approximation of the original discrete-time dynamics under learning. We characterize escape dynamics by analytically deriving the most probable escape point and mean escape time. The approximation is tested on the Phelps problem of a government controlling inflation while adaptively learning a misspecified Phillips curve, studied previously by Sargent (1999) and Cho et al. (2002) (henceforth, CWS), among others. We compare our results with simulations extended to very low values of the constant gain and show that, for the lowest gains, our approach approximates simulations relatively well. We express reservations regarding the applicability of any approach based on large deviations theory to characterizing escape dynamics for economically plausible values of constant gain in the model of CWS when escapes are not rare. We show that for these values of the gain it is possible to derive first passage times for learning dynamics reduced to one dimension without resort to large deviations theory. This procedure delivers mean escape time results that fit the simulations closely. We explain inapplicability of large deviations theory by insufficient averaging near the point of self-confirming equilibrium for relatively large gains which makes escapes relatively frequent, suggest the changes which might help approaches based on the theory to work better in this gain interval, and describe a simple heuristic method for determining the range of constant gain values for which large deviations theory could be applicable.

1. Introduction

The aim of this paper is to extend a continuous-time approximate approach to the analysis of escape dynamics in economic models with adaptive learning. We further test the approach on the Phelps problem of a government controlling inflation while adaptively learning the misspecified Phillips curve, studied previously by Sargent (1999), Cho et al. (2002), Sargent and Williams (2005), Sargent et al. (2006), McGough (2006), Ellison and Yates (2007), Williams (2009), Barnett and Ellison (2012), and Berardi (2013). The introduction of continuous-time approximation into analysis is motivated by the

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computational intensity of the approach used to derive theoretical characteristics of escape dynamics in the recent economic literature. Theoretical analysis of escape dynamics in economic models with adaptive learning allows analytical characterization of diverse economic phenomena such as currency crises, inflation episodes, endogenous collusion in oligopoly, and cycles of economic activity, see Cho and Kasa (2008, 2012), Williams (2002, 2003, 2004), Bullard and Cho (2005), Cho et al. (2002), and Kasa (2004). Escape dynamics has also been used to study large mutations in evolutionary games, see Kandori et al. (1993) and Binmore and Samuelson (1997).

These phenomena were modeled as a result of escape dynamics in economic models with boundedly rational economic agents. The agents use adaptive learning in a form summarized in Evans and Honkapohja (2001) to update their beliefs about model parameters. Among the papers devoted to this form of adaptive learning are Bray (1982), Bray and Savin (1986), Fourgeaud et al. (1986), Marcet and Sargent (1989), Evans and Honkapohja (1994a, 1994b, 1995), Marimon (1997), and many others. In this literature, agents are considered as econometricians who estimate forecasting models using standard statistical procedures, such as recursive least squares, stochastic gradient, or Bayesian learning, and who form beliefs about model parameters. The beliefs thus formed are then used to generate the actions of agents, and thus influence the realized values of economic variables which are taken as a new data point by the agents. In the next period, the agents update their beliefs with the new data. New beliefs then affect actions and economic variables, and this process repeats period after period. Under some regularity conditions, beliefs could converge to values corresponding to one of the rational expectations equilibria (REE) of the model. Stability under adaptive learning which guarantees such a convergence has been considered a very important characteristic of the REE in the recent monetary policy literature, c.f., Evans and Honkapohja (2003) or Bullard and Mitra (2002).

Beyond using adaptive learning as a *de facto* equilibrium selection mechanism or a tool for designing policy rules, one could concentrate on the dynamics of the model under adaptive learning as such, in particular, in a case of adaptive learning with a constant gain.[1] In this case, convergence of the learning process to REE is only in distribution: there are persistent fluctuations around the REE caused by such learning, and thus rare events—large distance movements called "escapes"—may occur with nonzero probability. During an escape, agents' beliefs about the model move further from nearly rational expectations. As a rule, their actions and the values of realized economic variables also deviate from those observed in the REE.

Analysis of escape dynamics caused by the adaptive learning process is possible using the theory of large deviations, cf., Freidlin and Wentzell (1998) (FW henceforth), Dupuis and Kushner (1989), and others. Depending on what version of large deviations theory—continuous-time by FW or discrete-time by Dupuis and Kushner (1989)—one wants to utilize, there are two possible approaches to the theoretical analysis of escape dynamics: the discrete-time approach and the continuous-time approach. The discrete-time approach, which has received wider attention in the literature, is based on the analytical derivation of escape dynamics for the original discrete-time Stochastic Recursive Algorithm (SRA) used to describe a learning process. In the continuous-time approach, a continuous-time diffusion approximation of the discrete-time SRA is derived, and then escape dynamics are studied for this approximation. Kasa (2004) applied this approach to a simple one-dimensional model. In this paper, we extend the approach to a multi-dimensional, non-reachable case, and concentrate on applying it to a linear continuous time approximation of the learning dynamics.

The discrete-time approach was used in the majority of the papers cited above, in particular in Cho et al. (2002) (henceforth CWS). These papers work directly with discrete-time SRAs and use the earlier results of Williams, who derived numerically the action functional for a linear-quadratic case when the state variable process is autoregressive with Gaussian noise. The basic problem associated with this approach is that characterizing escape dynamics for the discrete-time process as proposed by CWS implies numerical calculation of a functional in a calculus-of-variation problem that leads to a system of nonlinear differential equations with numerically derived right hand side functions. For complicated problems (with many lags and/or high dimensionality), this approach can become numerically intractable. An analytical solution for escape dynamics of a discrete-time process can be derived only for a restrictive class of learning processes, such as recursive least squares or stochastic gradient learning with a constant gain with Gaussian shocks.

The continuous-time approximation proposed in our paper contributes to a partial resolution of this problem. Our approximation around the REE is a linear diffusion with constant coefficients. In large deviations theory, all escape dynamics characteristics such as the expected time until the beliefs escape any given neighborhood D of the REE, the point through which this escape is most likely, and the probability of leaving D within a given amount of time, are obtained by minimizing a so-called action functional on the boundary of the neighborhood, ∂D. Given our choice of the approximating diffusion, this task is a standard linear control theory problem: minimizing the action functional is equivalent to finding a minimum of a quadratic form on ∂D, where a closed form solution for many geometric forms of boundaries exists. We argue that our approach allows the construction of an approximation to the true characteristics of escape dynamics, which would be hard to derive otherwise, and we discuss settings in which this approximation delivers useful results.

In order to compare the performance of the two approaches for deriving escape-dynamics characteristics, the continuous-time approximation is tested on a model in which the escape dynamic characteristics were already derived using the discrete-time approach: the Phelps problem of a government controlling inflation while adaptively learning the approximate Phillips curve, studied previously by Sargent (1999), CWS, and others. In contrast to most of the previous

[1] Constant gain learning discounts the past by assigning greater weight to more recent data.