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Biased Bayesian learning with an application to the risk-free rate puzzle

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ABSTRACT

Based on the axiomatic framework of Choquet decision theory, we develop a closed-form model of Bayesian learning with ambiguous beliefs about the mean of a normal distribution. In contrast to rational models of Bayesian learning the resulting Choquet Bayesian estimator results in a long-run bias that reflects the agent's ambiguity attitudes. By calibrating the standard equilibrium conditions of the consumption based asset pricing model we illustrate that our approach contributes towards a resolution of the risk-free rate puzzle. For a plausible parameterization we obtain a risk-free rate in the range of 3.5–5%. This is 1–2.5% closer to the empirical risk-free rate than according calibrations of the rational expectations model.

1. Introduction

Starting with the seminal contribution of Lucas (1978), the consumption based asset pricing model for a representative consumer economy has become the workhorse of the macroeconomic finance literature. As its main virtue the model derives the relationship between the consumer's uncertainty with respect to future consumption growth and equilibrium asset prices. However, as first demonstrated by Mehra and Prescott (1985), predicted asset returns under the rational expectations hypothesis are, by some large margin, at odds with actually observed asset returns. Several authors—e.g., Cecchetti et al. (2000), Brav and Heaton (2002), Abel (2002), Giordani and Soderlind (2006), Jouini and Napp (2008)—have relaxed the rational expectation hypothesis in order to explain asset-pricing puzzles through "incorrect" subjective beliefs. Any attempt to relax the rational expectations hypothesis in this manner faces the following questions:

Question 1: Why should subjective beliefs significantly differ from objective probabilities?
Question 2: Even if there exist initial differences between subjective beliefs and objective probabilities, why do these differences not vanish through Bayesian learning from large data-samples?

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Question 3: Even if any significant difference between subjective beliefs and objective probabilities is persistent, can plausible magnitudes of such differences explain observed asset-pricing puzzles?

We develop a closed-form model of biased Bayesian learning of the mean of a normal distribution which addresses the above questions on a sound decision-theoretic foundation. As our starting point we modify a standard closed-form model of Bayesian learning—according to which a decision maker has a normal distribution as prior over the unknown mean parameter of some normal distribution—by two more realistic assumptions. First, the prior is given as a truncated normal distribution. Second, information is given as interval- rather than point-observations. We then embed this benchmark model of rational Bayesian learning within a model of Choquet Bayesian learning. This allows us to capture an agent's ambiguity attitudes through neo-additive capacities in the sense of Chateauneuf et al. (2007). The agent of our model resolves this ambiguity by putting additional decision-weight on extreme (zero versus one) probabilities. As a consequence, the Choquet Bayesian estimator will generically result in an under- or over-estimation compared to the corresponding rational Bayesian estimator.

In the remainder of this introduction we sketch in more detail how our approach addresses Questions 1–3.

1.1. Addressing Question 1: Neo-additive capacities

We model subjective beliefs as non-additive probability measures such as neo-additive capacities. These arise as generalizations of subjective additive probability measures in Choquet expected utility (CEU) theory, which relaxes Savage's (1954) sure thing principle in order to accommodate for ambiguity attitudes as elicited in Ellsberg (1961) paradoxes (Schmeidler, 1986, 1989; Gilboa, 1987). CEU theory is formally equivalent to (cumulative) prospect theory (=PT 1992) (Tversky and Kahneman, 1992; Wakker and Tversky, 1993) whenever PT 1992 is restricted to gains (for a comprehensive treatment of this equivalence see Wakker, 2010). PT 1992, in turn, extends the celebrated concept of original prospect theory by Kahneman and Tversky (1979) to the case of several possible gain values in a way that satisfies first-order stochastic dominance.

By focusing on neo-additive capacities in the sense of Chateauneuf et al. (2007) we model the possible difference between ambiguous subjective beliefs and objective probability measures in a parsimonious way. In addition to information about the sample mean the agent's estimator of the distribution's mean reflects ambiguity. This is expressed through the parameter $\delta \in [0,1]$. Ambiguity is then resolved through the agent's ambiguity attitudes, resulting either in under- or overestimation. This is expressed through an “optimism” parameter $\lambda \in [0,1]$. Our model of Choquet Bayesian learning thus considers two additional parameters that are rooted in axiomatic decision-theory. It nests the benchmark model of rational Bayesian learning ($\delta = 0$).

To work with neo-additive capacities is attractive for two reasons. First, neo-additive capacities reduce the potential complexity of non-additive probability measures in a very tractable way. Important empirical features, e.g., inversely S-shaped probability transformation functions are thereby portrayed (Wakker, 2010, Chapter 11).[1] Second, Bayesian learning with respect to neo-additive capacities is closed, i.e., the prior and posterior are conjugate distributions.

1.2. Addressing Question 2: biased Bayesian learning

Standard models of consistent (=rational) Bayesian learning have been applied to several topics in economics and management sciences. For instance, early contributions by Cyert et al. (1978) and Tonks (1983) apply a Bayesian learning model—formulated within a normal distribution framework—to a firm's decision problem to invest in different technological processes. Viscusi (1979) and Viscusi and O'Connor (1984) apply a Bayesian learning model—formulated within a Beta-distribution framework—to the risk-learning behavior of workers in injury-prone industries. Viscusi (1990, 1991) uses the same learning model to address the question of how far smokers underestimate their health risk.

Zimper (2009) and Zimper and Ludwig (2009) have developed Choquet Bayesian learning models as generalizations of Viscusi's Beta-distribution approach to non-additive beliefs. In contrast, the present paper's Choquet Bayesian learning model is based on a normal distribution framework similar to Cyert et al. (1978) and Tonks (1983). As one main difference, the rational benchmark model of our approach takes a truncated normal distribution as prior. As a second main difference, we consider updating conditional on imprecise information; that is, we formally define information as interval- rather than as point-observations. These assumptions turn out to be technically and conceptually convenient when we extend the rational benchmark model to our model of Choquet Bayesian learning. They also add more realistic appeal to the rational benchmark model. For example, a representative agent's realistic prior over the mean of the annual consumption growth rate—which will be relevant to the risk-free rate puzzle discussed in this paper—should arguably assign positive probability mass only to some subset $(-\infty < a, b < \infty)$ and not to any subset of the $(-\infty, \infty)$ interval.

The Choquet Bayesian estimator of our model remains generically bounded away from the sample mean whenever the agent expresses the slightest initial ambiguity about his prior. This contrasts with consistency results for Bayesian estimators for additive probability spaces (Doob, 1949). Consequently, our approach provides a sound decision-theoretic answer to Question 2 as to why an initial difference between subjective beliefs and objective probabilities may, actually, increase rather than decrease in the long-run.

[1] To quote Peter Wakker (2010): “[…] the neo-additive functions are among the most promising candidates regarding the optimal tradeoff of parsimony and fit.” (p. 209).