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Bounded interest rate feedback rules in continuous-time

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ABSTRACT

This paper analyzes the dynamic consequences of interest rate feedback rules in a flexible-price model where money enters the utility function. Two alternative rules are considered based on past or predicted inflation rates. The main feature is to consider inflation rates that are selected over a bounded time horizon. We prove that if the Central Bank's forecast horizon is not too long, an active and forward-looking monetary policy is not destabilizing: the equilibrium trajectory is unique and monotonic. This is an advantage with respect to active and backward-looking policies that are shown to lead to a unique but fluctuating dynamic.

1. Introduction

Since McCallum (1981), monetary feedback rules have been studied to reestablish determinacy in monetary models (Bernanke and Woodford, 1997, Woodford, 2003). The choices pertaining to the modeling of the variables' timing are particularly important in those models (Carlstrom and Fuerst, 2000, 2001), and recommendations may vary depending on whether a discrete- or continuous-time representation is chosen (Dupor, 2001; Carlstrom and Fuerst, 2003, 2005). In this paper, we study and extend a framework initially proposed by Benhabib (2004) that mix both representations.

This is a continuous-time model with flexible prices and money in the utility function where the current nominal interest rates are set by a Central Bank according to a feedback rule, as in Benhabib et al. (2001). The main feature is that interest rate policy is neither defined according to the current value of inflation, nor on its value over an infinite horizon, but on its values over a finite horizon. With this assumption, the continuous-time model is more similar to traditional discrete-time models used in the literature. It turns out to be easier to solve. As recalled by Benhabib (2004), with discrete-time frameworks, the order of the difference equation that describes the equilibrium increases with the number of lagged inflation rates. For instance, the Taylor rule based on inflation recorded over the last four quarters (Taylor, 1993) implies that dynamics are described by an equation of at least 4th order, which is difficult to study analytically. In continuous-time, considering a bounded backward-looking rule leads to a dynamic that is described by a delay differential equation. We show how to use recent mathematical results on functional differential equations (d'Albis et al., 2012, 2013) to easily solve analytically the

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issue of the determinacy of the equilibrium in the framework developed by Benhabib (2004). We also extend his analysis by considering a forward-looking rule that leads to an advance differential equation. According to Clarida et al. (1998, 2000), forward-looking rules seem to be more realistic while Orphanides (2001) shows that they provide a better fit of real time data than current or backward-looking rules. However, advance differential equations are dramatically different from delay differential equations. The former are characterized by an unstable manifold of infinite dimensions whereas it is the stable manifold of these latter that are of infinite dimensions. Moreover, the status of initial variables may change and the respective roles of backward and forward variables in the dynamics are different. In d'Albis et al. (2013), we provide distinct theorems that permit to establish the determinacy properties in both configurations. By applying them, we are able to compare backward-looking and forward-looking rules in the same framework.

The results of our study, where consumption and money are assumed to be complementary in the utility function[1], are as follows. When the interest rate rule is a function of past inflation rates, equilibrium is indeterminate if the policy is passive and unique if the policy is active. This result holds whatever the length of the horizon for which inflation rates are taken into account in the interest rate rule. However, even when the equilibrium is determinate, the dynamics are characterized by short-term fluctuations that vanish when the backward horizon of the Central Bank is infinite. Bounding the backward horizon of the Central Bank creates some dependency to a specific initial trajectory that influences the dynamics through overreactions of the nominal interest rate. Those short run fluctuations disappear when the Central Bank does not bound its backward horizon and uses all available information on past inflation rates. When the interest rate rule is a function of future inflation rates, equilibrium remains indeterminate if the policy is passive; on the other hand, if the policy is active, equilibrium is unique provided that the Central Bank's forecast horizon is not too distant. Forward-looking feedback rules are known to be destabilizing (Woodford, 1999; Benhabib et al., 2001 or Bernanke and Woodford, 1997) and require aggressive interest rates policy to guaranty the determinacy of the equilibrium. By limiting the forecast horizon of the Central Bank, one may obtain the same result. This drastically reduces the set of equilibria compatible with perfect expectations to a unique trajectory. This result complements earlier findings from alternative monetary models. In a New Keynesian sticky-price model Batini and Pearlman (2002) and Batini et al. (2004, 2006) show that indeterminacy occurs if the forecast horizon lies too far into the future. To establish their results, they use a perturbation method, based on the particular form of the characteristic polynomial, but have to restrict to a very particular feedback rule that depends on expectations on the inflation rate that will prevail T periods ahead. We therefore consider a more general rule that take into account all inflation rates over a bounded interval. Moreover, sticky-price models may be different from the flexible price ones. For instance, when the Central Bank's forecast horizon is unbounded, we find that the rule is more prone to indeterminacy whereas Levine et al. (2007) find the contrary in a sticky-price model.

We recognize certain limitations to our study. First of all, we do not investigate the global dynamics of the system despite the fact that several studies have demonstrated its importance in interest rate policies (Benhabib et al., 2003; Eusepi, 2005; Cochrane, 2011). Similarly, we do not study permanent oscillations, especially those generated by Hopf bifurcations, despite the fact they may appear in this approach. In both instances, we are limited by the fact that there are no general theorems for the type of equations being considered.

In Section 2, we present the model whose solution is studied in Sections 3 and 4, where we make the distinction between backward-looking and forward-looking policies. Our conclusions are discussed in Section 5.

2. Feedback rules over a bounded horizon

We consider a model that is similar to those studied by Benhabib et al. (2001) and Benhabib (2004). This is a flexible-price model where nominal interest rates are set by the Central Bank as a function of past or forecasted inflation rates. The novelty is to consider that backward and forward horizons of the Central Bank are bounded.

Time is continuous and is denoted by $t \in \mathbb{R}_+$. Let $c(t)$, $m(t)$ and $a(t)$ be respectively real consumption, real balances held for non-production purposes and real financial wealth. The household's problem is

$$\max_{\{c,m,a\}} \int_0^{\infty} e^{-rt} U(c(t), m(t))\, dt$$
$$\text{s.t.} \quad \left| \begin{array}{l} a'(t) = [R(t)-\pi(t)]a(t) - R(t)m(t) + Y - c(t) - \tau(t), \\ a(0) > 0 \text{ given}, \\ \lim_{t \to +\infty} a(t) e^{-\int_0^t [R(z)-\pi(z)]\, dz} \geq 0, \end{array} \right. \tag{1}$$

where $r > 0$ and $Y > 0$ denote the rate of time preference and the output respectively. $R(t)$, $\pi(t)$ and $\tau(t)$ are perfectly anticipated by the household and denote the trajectories of the nominal interest rate, the inflation rate and the real lump-sum taxes respectively. The instant utility function $U(c(t), m(t))$ is strictly increasing ($U_c > 0, U_m > 0$) and strictly concave ($U_{cc} > 0, U_{mm} > 0$) in both arguments. Moreover, consumption and real balances are assumed to be complementary ($U_{cm} > 0$), which also implies that they are both normal goods. The cases where real balances are substitutable with

[1] Assumptions such that output can be produced with money or that money and consumption are complements in the utility function constitute immediate extensions that we did not considered in order focusing on the role of bounded horizon in feedback rules.