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Characterization of a risk sharing contract with one-sided commitment

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ABSTRACT

In this paper I provide a stopping-time-based solution to a long-term contracting problem between a risk-neutral principal and a risk-averse agent. The agent faces a stochastic income stream and cannot commit to the long-term contracting relationship. To compute the optimal contract, I also design an algorithm that is more efficient than value-function iteration.

1. Introduction

The theory of contracting with limited commitment has been applied to study a wide variety of economic issues, including asset pricing (cf. Kehoe and Levine, 1993; Alvarez and Jermann, 2000), consumption inequality (cf. Krueger and Perri, 2006), and the welfare effects of a progressive tax (cf. Krueger and Perri, 2011). The standard approach to solving these contracting problems is to iterate on the principal's value function.[1] However, value-function iteration (VFI) provides little general analytical characterization; further, when the discount factor is close to one, the value function converges slowly, making it computationally inefficient. The main contribution of this paper is to provide a constructive stopping-time-based procedure for solving the optimal contract with one-sided commitment. This method fully reveals the risk sharing dynamics in the contract. Moreover, I design a stopping-time-based algorithm that is two orders of magnitude faster than value-function iteration.

My model features a risk-neutral fully committed principal and a risk-averse noncommitted agent, and generalizes Ljungqvist and Sargent (2004, Chapter 19) along three dimensions. While they assume that the agent's income is independently and identically distributed (i.i.d.), the outside option is autarky, and the principal and the agent discount

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[1] Relevant aspects of the agent's history are first summarized in a single variable, which is the promised utility to the agent. Then the contracting problem is transformed into a dynamic programming problem, and recursive techniques are applied to solve the problem (cf. Spear and Srivastava, 1987; Abreu et al., 1990).

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the future at a common rate, I allow for a Markov-chain income process, an arbitrary outside option, and different discount rates. The three generalizations in my model are motivated by the following observations. First, it is well documented that people experience large and persistent income shocks over the life cycle. The quantitative features of the income process are poorly approximated by i.i.d. shocks. Second, agents in a number of long-term relationships have outside options better than autarky. For instance, in wage contracting between a firm and a worker, the worker has the option to quit the current job and find a new one. Last, I allow for different discount rates because when the principal in the model takes the interpretation of a financial intermediary, his discount factor should be determined by the interest rate. In a general equilibrium model, the endogenously determined interest rate is typically lower than the reciprocal of the agent's discount factor.

In the optimal contract, the agent's consumption follows a simple recursive rule: consumption deviates each period from the first-best level by the smallest amount necessary to bring it above some (state-dependent) minimum level. Because the recursive rule is relatively easy to verify, my paper focuses on finding the minimum levels. I first solve a stopping-time optimization problem: the moment when the participation constraint binds is a stopping time, and the stopping time is chosen to minimize the agent's consumption flow before it arrives. Then I guess and verify that the minimum level is the minimized consumption flow in the above problem.

My characterization of the contract is related to the solutions in Ljungqvist and Sargent (2004, Section 19.3.3), Krueger and Uhlig (2006, Section 3.5), Thomas and Worrall (2007, Section 3.2) and Krueger and Perri (2011, Section 4). Similar to my stopping-time approach, their solutions do not rely on value-function iteration. However, they assume i.i.d. incomes and rely on the monotonic mapping between incomes and minimum consumption levels. Broer (2009, 2011) extend the methods of Krueger and Uhlig (2006) to the Markov case and provides a sufficient condition under which the mapping between incomes and minimum consumption levels is monotone. However, his sufficient condition is violated for empirically relevant income processes such as the one calibrated by Krueger and Perri (2006). By contrast, my stopping-time approach does not depend on any particulars of the income process or the ordering of minimum consumption levels.

Stopping-time approaches have been used in continuous-time models. For instance, in liquidity constraint models in finance, Detemple and Serrat (2003) show that the optimal consumption portfolio problem of an individual is equivalent to a stopping-time problem in which wealth is optimally allocated over a random time period, during which the individual is not constrained. Grochulski and Zhang (2011) study a contracting problem in which the agent's income follows a geometric Brownian motion. Their analysis relies heavily on the fact that the stopping time is generated by a Brownian motion, hence is of limited value in other contexts. This paper allows for any Markov-chain income process and is, to the best of my knowledge, the first that applies stopping-time techniques to a discrete-time limited-commitment problem.

The recursive rule in this paper is related to a similar rule in two-sided limited-commitment models (cf. Thomas and Worrall, 1988; Ljungqvist and Sargent, 2004, Chapter 20). Because neither the principal nor the agent can commit in their models, both a minimum and a maximum level exist for the agent's consumption in each state. Their proof, however, is not constructive; therefore, to obtain these endogenous minimum and maximum consumption levels, they still need to iterate on value functions. By contrast, I study only one-sided limited commitment, but I am able to analytically construct the minimum consumption levels. Thus, this paper solves completely, in the context of one-sided commitment, the fundamental problem concerning risk sharing dynamics.

This paper is also consistent with the findings in Ray (2002) and Krueger and Uhlig (2006). When the principal and the agent are equally patient, the agent's continuation utility in the long run will be sufficiently high so that the participation constraint no longer binds. When the principal is more patient, the agent's consumption has a downward drift, and the agent's participation constraint binds even in the long run. The model then predicts a nontrivial stationary distribution of consumption.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 uses an example to motivate the general result. Section 4 presents the stopping-time characterization of the optimal contract. In Section 5, I design an efficient algorithm to compute the minimum consumption levels. This algorithm does not involve VFI and terminates in finite steps. Section 6 discusses extensions and limitations of the model. The proofs of all the results in the paper are provided in an appendix.

2. A risk sharing problem

Consider a risk-neutral principal and a risk-averse agent who engage in long-term contracting at time 0. Time is discrete and infinite. Preferences of the agent are represented by the expected utility function

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right],$$

where c_t is the agent's consumption at time t, $\beta \in (0,1)$ is his discount factor and E is the expectation operator. I make the following assumption on the utility function:

Assumption 1.

(i) $u:\mathbb{R}_{++}\to\mathbb{R}$ is twice continuously differentiable, $u'>0$, and $u''<0$.