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New insights into optimal control of nonlinear dynamic econometric models: Application of a heuristic approach

D. Blueschke[a], V. Blueschke-Nikolaeva[a], I. Savin[b,*]

[a] University of Klagenfurt, Austria
[b] DFG Research Training Program 'The Economics of Innovative Change', Friedrich Schiller University Jena and the Max Planck Institute of Economics, Bachstrasse 18k Room 216, D-07743 Jena, Germany

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ABSTRACT

Optimal control of dynamic econometric models has a wide variety of applications including economic policy relevant issues. There are several algorithms extending the basic case of a linear-quadratic optimization and taking nonlinearity and stochastics into account, but being still limited in a variety of ways, e.g., symmetry of the objective function and identical data frequencies of control variables. To overcome these problems, an alternative approach based on heuristics is suggested. To this end, we apply a 'classical' algorithm (OPTCON) and a heuristic approach (Differential Evolution) to three different econometric models and compare their performance. In this paper we consider scenarios of symmetric and asymmetric quadratic objective functions. Results provide a strong support for the heuristic approach encouraging its further application to optimum control problems.

1. Introduction

In many areas of science from engineering to economics, determining the optimal way of controlling a system is required in a great number of applications. In economics, one frequently asked question is how a policy maker should choose appropriate values for given controls, such as taxes or public consumption in order to, e.g., increase the growth rate of GDP, decrease unemployment rate or achieve other targets. In this case, calculation of the targeted state variables is restricted by a system of equations representing an econometric model of the country of interest.

Solving such an optimum control problem for nonlinear econometric models is the core of this paper. To this end, two different methods are considered, namely the OPTCON algorithm (Matulka and Neck, 1992; Blueschke-Nikolaeva et al., 2012), where classical techniques of linear-quadratic optimization are used, and Differential Evolution (DE, Storn and Price, 1997), which is a population based stochastic optimization method. Among DE's main advantages are the ability to explore complex search spaces with multiple local minima thanks to cooperation and competition of individual solutions in the DE's population, and the application easiness as it needs little parameter tuning (Maringer, 2008). The non-heuristic

* Corresponding author. Tel.: +49 3641 943275; fax: +49 3641 943202.
E-mail addresses: dmitri.blueschke@aau.at (D. Blueschke), vbluesch@aau.at (V. Blueschke-Nikolaeva), Ivan.Savin@uni-jena.de (I. Savin).

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approach, the OPTCON algorithm, on the other hand, is a more reliable and fast instrument for solving optimum control problems in standard applications.

However, like nearly all 'classical' methods, the OPTCON algorithm has several limitations. One, which is sometimes criticized in literature, is the required symmetry of the objective function. For the problems considered in this paper, the objective function is given in quadratic tracking form and equally penalizes positive and negative deviations from the given target values. In many situations, however, incorporation of different penalizing procedures for positive and negative deviations (in form of additional weighting coefficients) or inclusion of some indifference intervals would be desirable. Whereas it is nearly impossible to allow for this extension in the classic algorithm, it can be achieved by using a heuristic approach.

Before approaching the case of an asymmetric objective function, one has to make sure that DE can deliver a 'good' solution to the basic case of an optimum control problem. To demonstrate this, DE and OPTCON are applied to three macroeconometric models (with a deterministic scenario) and the performance of the two strategies is compared. Both methods are implemented in Matlab 7.11 to simplify their comparison. Due to the stochastic nature of DE and resulting need for several restarts of the strategy, a higher computational time is expected. For this reason, several possibilities to increase DE computational efficiency are also discussed.

Once the applicability of the heuristic approach has been demonstrated for the basic problem, it is extended and applied to solve the optimum control problem with an asymmetric objective function to three macroeconometric models. To this end, certain thresholds around the target values are introduced, inside which the objective function can be handled differently for positive and negative deviations. The resulting changes in the solutions are carefully analyzed and discussed both from the technical and economical perspectives.

The paper proceeds as follows. In Section 2 we define the class of problems to be tackled by the algorithms and describe the limitations, which are present in the OPTCON algorithm and are typical for 'classical' optimization methods. Section 3 briefly reviews the OPTCON algorithm as a classical approach and introduces DE as an alternative heuristic strategy. In Section 4 we analyze simulation results obtained for the two approaches with symmetric objective functions and extend DE to the asymmetric objective function scenario, testing the two strategies based on three econometric models (SLOVNL, SLOPOL4 and SLOPOL8). Section 5 concludes with a summary of the main findings and an outlook to further research.

2. Theoretical background

2.1. Type of problems

The task is to solve an optimum control problem with a quadratic objective function (a loss function to be minimized) and a nonlinear multivariate discrete-time dynamic system under additive and parameter uncertainties. The intertemporal objective function is formulated in quadratic tracking form, which is often used in applications of optimal control theory to econometric models. It can be written as

$$J = E\left[\sum_{t=1}^{T} L_t(x_t, u_t)\right] \tag{1}$$

with

$$L_t(x_t, u_t) = \frac{1}{2}\begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix}' W_t \begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix}, \tag{2}$$

where x_t is an n-dimensional vector of state variables that describes the state of the economic system at any point in time t, u_t is an m-dimensional vector of control variables, $\tilde{x}_t \in R^n$ and $\tilde{u}_t \in R^m$ are given 'ideal' (desired, target) levels of the state and control variables, respectively. T denotes the terminal time period of the finite planning horizon. W_t is an $((n+m) \times (n+m))$ matrix specifying the relative weights of the state and control variables in the objective function. The W_t matrix may also include a discount factor α, $W_t = \alpha^{t-1} W$. W_t (or W) is symmetric.

The dynamic system of nonlinear difference equations has the form

$$x_t = f(x_{t-1}, x_t, u_t, \theta, z_t) + \varepsilon_t, \quad t = 1, \ldots, T, \tag{3}$$

where θ is a p-dimensional vector of parameters that is assumed to be constant but unknown to the policy maker (parameter uncertainty), z_t denotes an l-dimensional vector of non-controlled exogenous variables, and ε_t is an n-dimensional vector of additive disturbances (system error). θ and ε_t are assumed to be independent random vectors with expectations, $\hat{\theta}$ and O_n, and covariance matrices, $\Sigma^{\theta\theta}$ and $\Sigma^{\varepsilon\varepsilon}$, respectively. f is a vector-valued function with $f^i(\ldots)$ representing the i-th component of $f(\ldots)$, $i = 1, \ldots, n$. Solving an optimum control problem means, therefore, to find a certain set of controls $(u_1^*, u_2^*, \ldots, u_T^*)$ which minimizes the objective function J, i.e. to find $u^* = \text{argmin}_u J$ with respect to (3).

For the study presented in this paper the deterministic case is considered only assuming the model parameters and the model equations to be exactly true. It means that parameters in θ are given without uncertainty and the error terms are zero. Applying a heuristic approach for stochastic case is a further research question and will be discussed in the future.