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On the long-run relationship between inflation and output in a spatial overlapping generations model ☆

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ABSTRACT

The paper builds a model that features spatial differentiation of markets, and then uses it to study, first, the relationship between inflation and the steady-state level of output, and second, the relationship between inflation and the steady-state distribution of output across the economy. A steady-state of the model entails a stationary distribution of money across the locations of the economy. With all else held fixed, a change in the rate of money-growth induces a change in the distribution of money, which leads to a change in labour supply and production throughout the economy. Thus the distribution of money provides a channel through which a change in monetary policy affects real economic activity.

1. Introduction

The paper studies the long-run relationship between real output and inflation in a monetary overlapping generations model in which markets are spatially differentiated. More specifically, an economy is endowed with a spatial structure in the form of a fixed graph such as those shown in Fig. 1. The vertices represent spatially distinct markets inhabited by two-period lived agents who produce in the first period of their lives and consume in the second period, but who consume only goods produced at neighbouring vertices. All transactions involve an exchange of goods for fiat money, and so currency travels from vertex to vertex as a consequence of production and exchange. A basic fact of the model is that the distribution of money across the locations of the economy is crucial in determining real economic activity. To be more precise, monetary policy is executed by means of lump sum transfers in each period to agents located at a subset of vertices, where the transfers occur according to a fixed rate of growth. Hence a monetary policy is formally characterized by two "parameters," one is the subset of vertices receiving transfers, and the other is the rate of growth in the money stock. To each monetary policy there exists a unique steady-state distribution of money across the vertices of the economy. Thus a change in the rate of money-growth will alter the steady-state distribution of money, and since relative prices depend on this distribution, it follows that a change in the rate of money growth will generate a change in both the overall level of output, and the distribution of output across the economy. Therefore, by acting upon the steady-state distribution of money, a change in the rate of money growth can have a long-run effect on both the level and distribution of output. Thus the main contribution of

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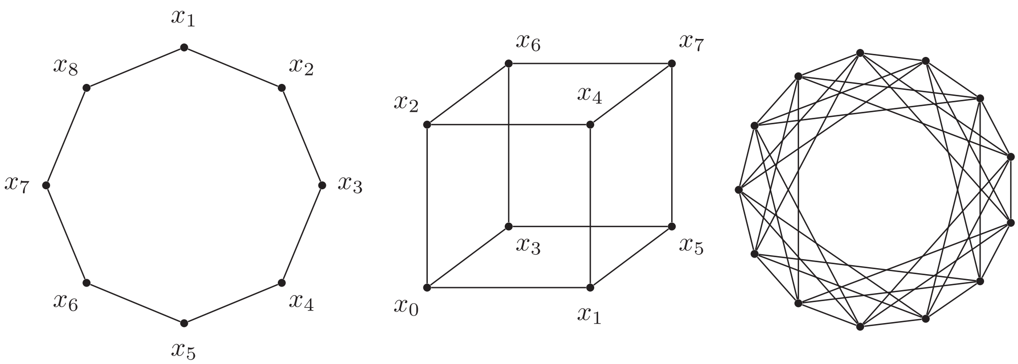

Fig. 1. The cyclic graph C_8 (left), the hypercube H_3 (middle), and the Paley graph P_{13} (right).

the paper is to show that there exists a role for the distribution of money as a channel through which changes in monetary policy affect real economic variables.

Hopefully the paper also makes a contribution to the class of tractable monetary models. The economies in the paper can be built on classes of graphs of arbitrary size and nearly endless variety, and yet all of the important economic variables admit closed-form solutions. The main consequence is to allow the distribution of money to interact with other economic variables in a tractable setting. This is in contrast to many important monetary models where the distribution of cash across heterogeneous agents is a complicating factor, which if not worked around, would severely limit the applicability of the underlying model. For instance, in his classic paper “The Optimum Quantity of Money” Milton Friedman (1969) proposes a thought experiment whereby money is dropped from a helicopter onto an economy at a steady rate. In framing the experiment, he is extremely careful to avoid the possibility that the distribution of cash would be altered through the agency of the helicopter, for such changes could affect the outcome of the experiment in ways that would be impossible to determine. Quite simply, Friedman's classic experiment would be vitiated by a loss of tractability if the distribution of currency were allowed to change. However, in this paper, changes in the distribution of currency are at the core of the analysis, and the fact that their impact can be studied is due entirely to the model's tractability.

Two additional examples are the important papers of Lagos and Wright (2005) and Shi (1997). Both of these modify the search-based model of Kyotaki and Wright (1989, 1993) so as to obtain a framework suitable for studying important questions in monetary theory. A key innovation in each paper is a clever modelling device that induces the distribution of money to collapse to a point mass at the close of each time period. Thus both papers provide examples of models where tractability becomes feasible only after the distribution of money is tamed.

Motivation for studying economies with a spatial structure is provided in part by certain ideas of Milton Friedman which are described in the opening paragraph of Section 1.1. Further motivation comes from recent work which shows that important segments of U.S. financial markets are endowed with a network structure. Soramaki et al. (2007) analyze the daily interbank payments over the Federal Reserve's Fedwire Funds Service (the Fedwire) by assuming that these payments flow through a graph. On a given day, any bank that either originates or receives a payment over the Fedwire is considered to be a vertex, and a directed edge leads from bank x to bank y if a payment originates at x and is received by y. They consider the graph generated by Fedwire payments made over the course of each day during the first quarter of 2004 (thus there is a total of 62 daily graphs). On an average day, the giant strongly connected component contains 5086 vertices and 76,614 directed edges.[1] A similar analysis is carried out by Bech and Atalay (2010) who use nearly 7 million Fedwire transactions during the period 1997–2006, to investigate the network structure of the overnight market for federal funds. On an average day in 2006, a daily graph would possess 470 vertices and 1543 directed edges.[2] Thus the “backbone” of the U.S. financial system is naturally represented by a large complicated graph. Moreover, these papers reflect a rapidly growing interest in using tools from graph theory to model modern economies.[3] (See Allen and Babus (2008) and Minoiu and Reyes (2011) for references.)

The main focus of the paper is the role of the distribution of money in shaping the relationship between inflation and both the level and distribution of output. To reiterate, to any monetary policy there corresponds a unique steady-state distribution of money, and, in general, any distribution of money will be associated with a distinct distribution of relative

[1] A *strongly connected component* of a directed graph is a maximal subset of vertices with the property that if x and y are any two elements of the subset, then there is a directed path going from x to y and another directed path going from y to x.

[2] Research along these lines is not restricted to the United States. Iori et al. (2008) perform a network analysis of the Italian segment of the European interbank market for overnight loans, and Minoiu and Reyes (2011) carry out a similar analysis on the network of banks engaged in cross-border lending.

[3] The motivation for pursuing this agenda is eloquently stated by Caballero (2010): “I suspect that embracing rather than fighting complexity and what it does to our modelling would help us make progress in understanding macroeconomic events. One of the weaknesses of the core stems from going too directly from statements about individuals to statements about the aggregate, where the main difference between the two comes from stylized aggregate constraints and trivial interactions, rather than from the richness and unpredictability of the linkages among the parts. We need to spend much more time modelling and understanding the topology of linkages among agents, markets, institutions, and countries.”