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First-order, buckling and post-buckling behaviour of GFRP pultruded beams. Part 2: Numerical simulation

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ABSTRACT

This work deals with the numerical evaluation of the structural response of simply supported (transversally loaded at mid-span) and cantilever (subjected to tip point loads) beams built from a commercial pultruded I-section GFRP profile. In particular, the paper addresses the beam (i) geometrically linear behaviour in service conditions, (ii) local and lateral-torsional buckling behaviour, and (iii) lateral-torsional post-buckling behaviour, including the effect of the load point of application location. The numerical results are obtained by means of (i) novel Generalised Beam Theory (GBT) beam finite element formulations, able to capture the influence of the load point of application, and (ii) shell finite element analyses carried out in the code Abaqus. These numerical results are compared with (i) the experimental values reported and discussed in the companion paper (Part 1) and (ii) values provided by analytical formulae available in the literature.

1. Introduction

Thin-walled pultruded composite profiles made of Glass Fibre Reinforced Polymers (GFRP) are being increasingly used in the construction industry, namely in corrosive environments or in applications requiring fast and light construction [1–7]. However, the mechanical properties exhibited by the GFRP material, namely the high strength-to-stiffness ratio and low shear modulus, render such profiles prone to several specific structural vulnerabilities – *e.g.*, brittle rupture, excessive deformations under service conditions or high susceptibility to local and global buckling phenomena. Furthermore, the scarce available specifications and/or guidelines for the design of GFRP members are either too conservative or incomplete [8–10], thus making the proper use of these profiles highly dependent on experimental studies and/or reliable numerical analysis tools.

It is well known that thin-walled beams subjected to major axis bending buckle globally in lateral-torsional modes and the corresponding post-buckling behaviour is only marginally stable, which means that the buckling moment/load may be viewed as an upper bound for the elastic strength – in other words, the (elastic) collapse of an initially imperfect beam involves large displacements and occurs for a moment/load close to its critical value. On the other hand, it is also widely recognised that the (vertical) distance from the point of load application to the cross-section shear centre strongly affects the beam buckling behaviour (buckling loads and mode shapes). Although this effect has been properly quantified in the context of steel beams for decades (*e.g.*, [11]), the same is not true for FRP composite beams – indeed, it was only a few years ago that Turvey [12] addressed this issue: he conducted an experimental, analytical and numerical investigation on the lateral-torsional buckling of FRP pultruded I-section cantilevers acted by point loads applied at the end section top flange, bottom flange and shear centre. It is still worth mentioning the numerical and experimental work carried out by Samanta and Kumar [13] – however, this work dealt only with distortional buckling. Mottram [14] adapted a formula to calculate critical (lateral-torsional) buckling moments in steel beams, included in the ENV (European Pre-Norm) version of Eurocode 3 [15], making it applicable to pultruded beams. He used appropriate elastic moduli, as proposed by Bauld and Tzeng [16], and compared the results with values yielded by the finite difference method. In the context of Generalised Beam Theory (GBT), Gonçalves and Camotim [17] were the first to propose a formulation able to capture the effect of the location of the load application point in steel beams. Quite recently, Silva et al. [18] developed a similar, but more general, approach that is also capable of capturing this effect in local, distortional and localised buckling analyses. The use of GBT to analyse the geometrically linear and buckling behaviour of composite FRP members is fairly recent and due to the research activity of Silvestre and Camotim [19–21] and Silva et al. [22].

Concerning local buckling in FRP members, several researchers conducted numerical and experimental investigations [23–29]. Kollár [23] developed simple analytical formulae to estimate the

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critical local buckling loads of orthotropic beams and columns with I and box cross-sections, which are based on plate solutions and take into account the rotational restraints due to the walls adjacent to the "critical wall", *i.e.*, that triggers the local buckling. These formulae provide rather accurate local critical buckling load estimates, when compared with other similar analytical tools [30].

Although there is work available on the use of GBT to analyse the elastic post-buckling behaviour of thin-walled members, almost all of it was carried out in the context of cold-formed steel members, which means that only *isotropic* materials were dealt with – aside from the pioneering work of Miosga [31], nowadays only of historical interest, it is worth mentioning the studies of (i) Silvestre and Camotim [32], who developed and implemented a beam finite element formulation to analyse geometrically imperfect members with unbranched open cross-sections,[1] (ii) Simão [35], who presented a Rayleigh–Ritz implementation of a formulation applicable to geometrically perfect members with arbitrary cross-sections, and (iii) Gonçalves [36], who developed (but did not implement) a formulation for geometrically imperfect members with arbitrary cross-sections. As far as orthotropic thin-walled members are concerned and to the authors best knowledge, the only GBT-based numerical post-buckling results available were obtained by Silva et al. in the context of FRP composite thin-walled columns with open [37] and arbitrary [38] cross-sections, based on a beam finite element implementation of a geometrically non-linear GBT formulation derived a few years ago by Silvestre [39].[2]

In Part 2 of this two-part paper, the GBT formulations mentioned in the previous paragraphs are extended in order to be able to capture the influence of the load application point location on the local and global buckling and post-buckling behaviours of FRP composite thin-walled members. Moreover, these formulations are based on a novel cross-section analysis approach recently developed by the authors [41]. Then, GBT-based finite element models are employed to analyse the first order, buckling and post-buckling behaviour of the GFRP pultruded I-section simply supported beams and cantilevers whose experimental tests were reported in the Part 1 of this paper [7]. One obtains (i) first order (linear) deformed configurations, (ii) critical moments/load values, together with the corresponding buckling mode shapes, and (iii) non-linear (post-buckling) equilibrium paths and deformed configurations. While the buckling results concern the (i) local and lateral-torsional buckling of a simply supported beam acted by a mid-span transverse point load and (ii) lateral-torsional buckling of a cantilever acted by a tip point load (the lateral-torsional buckling results are associated with point loads applied at the beam/cantilever top flange, bottom flange and shear centre), only cantilever (lateral-torsional) post-buckling analyses are carried out – it is worth pointing out that the latter are still preliminary results of an on-going investigation on the development and implementation of a non-linear GBT formulation for FRP composite members (see [42]).[3] For validation purposes, the GBT-based first-order, buckling and post-buckling results are compared with (i) the experimental data presented in the companion paper (Part 1) and also with (ii) values (ii_1) yielded by shell finite element analyses carried out in the code Abaqus [43] and (ii_2) provided by analytical formulae developed by Mottram [14] (lateral-torsional buckling) and Kollár [23] (local buckling).

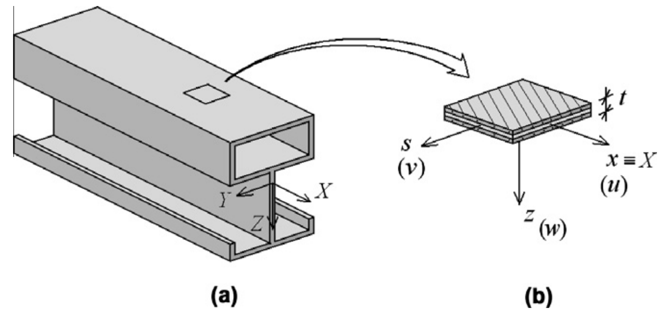

Fig. 1. (a) Arbitrary laminated FRP thin-walled prismatic member and (b) wall element, together with the corresponding local coordinate system.

2. Generalised Beam Theory

2.1. Geometrically non-linear formulation

Consider the supposedly arbitrary laminated FRP thin-walled prismatic member shown in Fig. 1(a), together with its global coordinate system X–Y–Z (X is the longitudinal axis). As for Fig. 1(b), it depicts a wall element and the corresponding local coordinate system x–s–z (x, s and z are oriented along the member longitudinal axis, cross-section mid-line and wall thickness) – u, v and w are the corresponding (local) displacement components.

In a GBT analysis, the displacement components of the member mid-surface are expressed as products of two functions, each depending on a single coordinate: either (i) s (the deformation mode shapes $u_k(s)$, $v_k(s)$ and $w_k(s)$) or (ii) x (the modal amplitude functions $\phi_k(x)$). Then, one has

$$u(x,s) = u_k(s)\phi_{k,x}(x) \quad v(x,s) = v_k(s)\phi_k(x) \quad w(x,s) = w_k(s)\phi_k(x), \tag{1}$$

where the comma indicates differentiation w.r.t. x. In order to incorporate the effect of the initial geometrical imperfections into the member post-buckling analysis, it is convenient to express them also in modal form, *i.e.*,

$$\bar{u}(x,s) = u_k(s)\bar{\phi}_{k,x}(x) \quad \bar{v}(x,s) = v_k(s)\bar{\phi}_k(x) \quad \bar{w}(x,s) = w_k(s)\bar{\phi}_k(x), \tag{2}$$

where functions $\bar{\phi}_k(x)$ provide the variation of the modal imperfection amplitudes along the member length. As in most thin-walled models, Kirchhoff's hypotheses are deemed valid (fibres initially normal to the plate mid-plane are inextensible and remain normal to that mid-plane after deformation – $\gamma_{xz} = \gamma_{sz} = \varepsilon_{zz} = 0$, *i.e.*, negligible wall bending transverse extensions and shear strains), thus meaning that ε_{xx}, ε_{ss}, γ_{xs} are the only strain components considered. One adopts here a simplified version of the Green–Lagrange strain–displacement relations, which include only the most relevant non-linear terms – those appearing in the engineering strain definition [37].[4] Taking into account these assumptions, the strain components read

[1] Very recently, Basaglia et al. [33,34] have extended this work to cover members with arbitrary open cross-sections and also the presence of localised displacement restraints (*e.g.*, those due to bracing).

[2] Some numerical results obtained by means of this formulation were reported by Silvestre and Camotim [40] – however, they were never published with sufficient detail.

[3] The GBT beam post-buckling results presented here are the first ones concerning FRP composite members under non-uniform bending and/or subjected to transverse loads applied off the cross-section shear centre. How to account adequately for the influence of these two aspects, which have already been mastered in the context of buckling analyses, is a topic of current investigation – this is why the above post-buckling results are termed "preliminary".

[4] It is again worth noting that the influence of this simplifying assumption on the post-buckling behaviour of FRP composite members (beams) under non-uniform bending and/or acted by transverse loads applied off the shear centre has never been investigated in the context of GBT-based analyses. As discussed in some detail in Section 3.5.2, it seems that additional non-linear terms will have to be included in the strain-displacement relations.