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Numerical solution of dynamic equilibrium models under Poisson uncertainty

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ABSTRACT

We propose a simple and powerful numerical algorithm to compute the transition process in continuous-time dynamic equilibrium models with rare events. In this paper we transform the dynamic system of stochastic differential equations into a system of functional differential equations of the retarded type. We apply the Waveform Relaxation algorithm, i.e., we provide a guess of the policy function and solve the resulting system of (deterministic) ordinary differential equations by standard techniques. For parametric restrictions, analytical solutions to the stochastic growth model and a novel solution to Lucas' endogenous growth model under Poisson uncertainty are used to compute the exact numerical error. We show how (potential) catastrophic events such as rare natural disasters substantially affect the economic decisions of households.

1. Introduction

The stochastic growth model in continuous time has received extensive study in the macro literature (following Merton, 1975; Chang and Malliaris, 1987).[1] This benchmark economy gave rise to the development of advanced models for capturing the main features of aggregate fluctuations, often referred to as dynamic stochastic general equilibrium (DSGE) models. These models are the workhorse in dynamic macroeconomic theory. We use them to organize our thoughts, interpret empirical data and for policy recommendations.

The literature on DSGE models, however, has been surprisingly quiet on the effects of large economic shocks such as natural disasters and economic and/or financial crises. Most of the papers focus on small and frequent 'business cycle shocks'. Therefore, departures from Normal uncertainty are largely unexplored. But the simple awareness of large and rare 'Poisson jumps' leads to an adjustment of households' optimal consumption plans. One crucial difference to business cycle shocks is that an econometrician may not observe rare events for a longer period, and thus households might appear to be irrational.

In economic theory, however, we use Poisson events to model, e.g., natural disasters (Barro, 2006), technological improvements (Wälde, 1999, 2005),[2] exploration for exhaustible resources (Quyen, 1991), and financial market bubbles

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[1] The discrete-time one-sector stochastic neoclassical model was pioneered by Brock and Mirman (1972). The mathematical theory of the neoclassical growth model has its origin in Ramsey (1928).

[2] Rare events in the form of Poisson uncertainty also form the basis in quality ladder and matching models (Grossman and Helpman, 1991; Aghion and Howitt, 1992; Lentz and Mortensen, 2008).

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(Miller and Weller, 1990). Similarly, from an empirical perspective, besides anecdotal catastrophic events such as the 2004 Sumatra–Andoman earthquake and tsunami (South Asia), the 2005 Hurricane Katrina (USA) and the recent 2011 Sendai earthquake (Japan), rare disasters are found to have substantial asset pricing and welfare implications (Barro, 2009). Moreover, there is empirical evidence for rare Poisson jumps (positive and negative) in US macro data (Posch, 2009).

For most applications, economists need to rely on numerical methods to compute the solutions to their models. Thus the literature is making a huge effort in developing powerful computational methods (cf. Judd, 1992; Judd and Guu, 1997). Unfortunately, no rigorous treatment of how to solve dynamic equilibrium models under Poisson uncertainty numerically has been provided so far, and the effects of rare events on approximation errors are unknown.[3]

This paper proposes a simple and powerful method for determining the transition process in dynamic equilibrium models under Poisson uncertainty numerically. It turns out that local approximation techniques are not applicable and most global numerical recipes need to account for the specific nature of rare events. We show how to extend existing standard algorithms when we allow for the possibility of rare events.

Our analysis builds on the continuous-time formulation of a stochastic neoclassical growth model based on Merton (1975). We use the continuous-time formulation for two reasons.[4] Firstly, we can easily compute stochastic differentials for transformations based on random variables under Poisson uncertainty. Secondly, for reasonable parametric restrictions we can solve the models by hand and obtain closed-form policy functions which can be used as a point of reference and to compute the exact numerical error.[5] From these benchmark solutions our numerical method is used to explore broader parameterizations. Our idea is to transform the system of stochastic differential equations (SDEs) into a system of functional differential equations of the retarded type (Hale, 1977). We apply the Waveform Relaxation algorithm, i.e., we provide a guess of the policy function and solve the resulting system of (deterministic) ordinary differential equations (ODEs) by standard techniques.

This procedure is applicable to models which imply a dynamic system of controlled SDEs under Poisson uncertainty. The controls are Markov controls in the form of policy functions (cf. Sennewald, 2007). Although our method can also be applied to Normal uncertainty, existing standard procedures can be used for this class of models (cf. Candler, 1999). We therefore do not advocate the use of the Waveform Relaxation algorithm over alternative approaches in all cases and applications. We aim at expanding the set of tools available to researchers by showing how to solve dynamic economies under Poisson uncertainty.

We show that our solution method works. Although the suggested procedure computes the policy functions for the complete state space — even for non-linear solutions — the maximum (absolute) error compared to the exact solutions is very small. A strength of our approach is that existing algorithms are easily extended to allow for Poisson uncertainty. We illustrate our approach for two popular methods computing numerical solutions to dynamic general equilibrium models, i.e., the backward integration (Brunner and Strulik, 2002) and the Relaxation algorithm (Trimborn et al., 2008). From an economic point of view, we find that (potential) large shocks affect optimal consumption and hours strategies.

The structure of the paper is as follows. In Section 2 of this paper, we describe the class of models of interest. In Section 3, we describe the Waveform Relaxation method in detail and discuss alternative approaches. In Section 4, we present two applications. The first is the stochastic growth model with rare disasters. We choose parameterizations that allow for analytical solutions to compute the numerical error. The second is the Lucas model of endogenous growth including a novel analytical solution under Poisson uncertainty. We conclude in Section 5.

2. The macroeconomic theory

This section introduces a broad class of economic models under Poisson uncertainty which can be solved by means of Waveform Relaxation. Our algorithm (presented in Section 3.2) can be used to study transitional dynamics in models under Poisson uncertainty. We show how standard numerical techniques, which compute the optimal time paths of variables, can be extended to allow for Poisson uncertainty, i.e., how they can be used to solve a system of (stochastic) differential equations. A discussion of alternative approaches is provided in Section 3.3. For this purpose, we develop our theoretical framework in Section 2.1, and then present a simple procedure to obtain the (optimal) dynamic system in Section 2.2.

Our motivation stems from the rare disaster literature (Rietz, 1988; Barro, 2006, 2009). Hence, our illustrations are mainly for rare events such as earthquakes or hurricanes which remove a certain fraction of the capital stock. Obviously, our framework is not limited to this particular class of models. For example, infrequent productivity increases are found in the endogenous growth literature (Wälde, 2005). In any case, below we demonstrate that models with rare (but potentially large) economic shocks are conceptionally different from models with smaller shocks, e.g., 'business cycle shocks' resulting from Normal uncertainty. In a nutshell, we show below that the Bellman equation for models under Poisson uncertainty is a

[3] Generally most numerical methods are highly accurate locally (cf. Taylor and Uhlig, 1990; Christiano and Fisher, 2000; Schmitt-Grohé and Uribe, 2004; Aruoba et al., 2006).

[4] Continuous time models under uncertainty are widely used in economics (for a survey see Wälde, 2011), a continuous-time New Keynesian model is in Fernández-Villaverde et al. (2011).

[5] Analytical solutions for parametric restrictions are frequently used in macro models (Turnovsky, 1993, 2000; Corsetti, 1997; Wälde, 2005, 2011; Turnovsky and Smith, 2006; Posch, 2009).