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Solving DSGE models with a nonlinear moving average

Hong Lan*, Alexander Meyer-Gohde[1]

Humboldt-Universität zu Berlin, Institut für Wirtschaftstheorie II, Spandauer Straße 1, 10178 Berlin, Germany

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ABSTRACT

We propose a nonlinear infinite moving average as an alternative to the standard state space policy function for solving nonlinear DSGE models. Perturbation of the nonlinear moving average policy function provides a direct mapping from a history of innovations to endogenous variables, decomposes the contributions from individual orders of uncertainty and nonlinearity, and enables familiar impulse response analysis in nonlinear settings. When the linear approximation is saddle stable and free of unit roots, higher order terms are likewise saddle stable and first order corrections for uncertainty are zero. We derive the third order approximation explicitly, examine the accuracy of the method using Euler equation tests, and compare with state space approximations.

1. Introduction

Solving models with a higher than first order degree of accuracy is an important challenge for DSGE analysis with the growing interest in nonlinearities. We introduce a novel policy function, the nonlinear infinite moving average, to perturbation analysis in dynamic macroeconomics. This direct mapping from shocks to endogenous variables neatly dissects the individual contributions of orders of nonlinearity and uncertainty to the impulse response functions (IRFs). For economists interested in studying the transmission of shocks, our method offers new insight into the propagation mechanism of nonlinear DSGE models.

The nonlinear moving average policy function chooses as its state variable basis the infinite history of past shocks.[2] The nonlinear DSGE perturbation literature initiated by Gaspar and Judd (1997), Judd and Guu (1997), and Judd (1998, Chapter 13) has thus far operated solely with state space methods.[3] Our infinite dimensional approach is longstanding in

* Corresponding author. Tel.: +49 30 2093 1466.
E-mail addresses: lanhong@cms.hu-berlin.de (H. Lan), alexander.meyer-gohde@wiwi.hu-berlin.de (A. Meyer-Gohde).
[1] Tel.: +49 30 2093 5720; fax: +49 30 2093 5696.

[2] Kalman's (1980) “external” or “empirical” approach to system theory in contrast to the “internal” or “state-variable” approach of the state space methods currently more familiar to DSGE practitioners. See Woodford (1986) for a theoretical foundation of nonlinear DSGE solutions in this space of infinite sequences of innovations.

[3] See Collard and Juillard (2001a, 2001b), Jin and Judd (2002), Schmitt-Grohé and Uribe (2004), Anderson et al. (2006), Lombardo and Sutherland (2007), and Kim et al. (2008). Recent work of Aruoba et al. (2012) links their quadratic autoregressive (QAR) time series model within a DSGE context to the Volterra series expansion that we use as our solution basis.

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linear models and delivers the same solution as state space methods for linear models.[4] For the nonlinear focus of this paper, however, it provides a different solution. Deriving the direct mapping from shocks to endogenous variables—a Volterra series expansion—facilitates familiar impulse response analysis and makes clear the caveats introduced by nonlinearity. These include history dependence, asymmetries, a breakdown of superposition and scale invariance, as well as harmonic distortion.[5]

As highlighted by Gomme and Klein (2011) in their second order approximation, deriving perturbation solutions with standard linear algebra increases the transparency of the technique and makes coding the method more straightforward. In that vein, we adapt Vetter's (1973) multidimensional calculus to provide a mechanical system of differentiation that maintains standard linear algebraic structures for arbitrarily high orders of approximation. We implement our approach numerically by providing an add-on for the popular Dynare package.[6] We then apply our method to the stochastic growth model of Aruoba et al. (2006) for comparability and explore the resulting decomposition of the contributing components of the responses of variables to exogenous shocks. We develop Euler equation error methods for our infinite dimensional policy function and confirm that our moving average solution produces approximations with a degree of accuracy comparable to state space solutions of the same order of approximation presented in Aruoba et al. (2006).[7]

We make two assumptions on the first order (i.e., linear) approximation: it is saddle stable and it is free of unit roots. The first is the standard Blanchard and Kahn (1980) assumption and we show that the resulting stability from the first order is passed on to higher order terms. The second ensures the boundedness of corrections to constants and the two together guarantee the local invertibility of a standard state space policy function to yield our infinite moving average.

The paper is organized as follows. The model and the nonlinear infinite moving average policy function are presented in Section 2. In Section 3, we develop the numerical perturbation of our nonlinear infinite moving average policy function explicitly out to the third order. We compare our policy function with state space policy functions in Section 4. We apply our method to a standard stochastic growth model in Section 5, a widely used baseline for numerical methods in macroeconomics. In Section 6, we develop Euler equation error methods for our infinite dimensional solution form and quantify the accuracy of our method. Section 7 concludes.

2. Problem statement and solution form

We begin by introducing our class of models, a standard system of (nonlinear) second order expectational difference equations. In contrast with the general practice in the literature, however, the solution will be a policy function that directly maps from realizations of the exogenous innovations to the endogenous variables of interest. We then approximate the solution with a Volterra series and present the matrix calculus used in subsequent sections.

2.1. Model class

We analyze a family of discrete-time rational expectations models given by

$$0 = E_t[f(y_{t-1}, y_t, y_{t+1}, u_t)], \quad \text{where } u_t = \sum_{i=0}^{\infty} N^i \varepsilon_{t-i} \tag{1}$$

f is an $(neq \times 1)$ vector valued function, continuously M-times (the order of approximation to be introduced subsequently) differentiable in all its arguments; y_t is an $(ny \times 1)$ vector of endogenous variables; the vector of exogenous variables u_t is of dimension $(nu \times 1)$ and it is assumed that there are as many equations as endogenous variables $(neq = ny)$. N is the $(nu \times nu)$ matrix of autoregressive coefficients of u_t, presented here in moving average form. The eigenvalues of N are assumed all inside the unit circle so that u_t admits this infinite moving average representation; and ε_t is an $(ne \times 1)$ vector of exogenous shocks of the same dimension $(nu = ne)$.[8] ε_t is assumed independently and identically distributed such that $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^{\otimes[n]})$ exists and is finite for all n up to and including the order of approximation to be introduced subsequently.[9]

As is usual in perturbation methods, we introduce an auxiliary parameter $\sigma \in [0,\ 1]$ to scale the uncertainty in the model. The value $\sigma = 1$ corresponds to the "true" stochastic model under study and $\sigma = 0$ represents the deterministic version of the

[4] Compare, e.g., the state space representations of Uhlig (1999), Klein (2000), or Sims (2001) with the infinite moving-average representations of Muth (1961), Whiteman (1983) or Taylor (1986).

[5] See also Priestly (1988), Koop et al. (1996), Potter (2000), and Gourieroux and Jasiak (2005).

[6] See Adjemian et al. (2011) for Dynare. Our add-on can be downloaded at http://www.wiwi.hu-berlin.de/professuren/vwl/wtm2/mitarbeiter/meyer-gohde.

[7] Aruoba et al. (2006) also explore several global methods (projection, value function iteration) and our choice allows comparability to these other methods. Our focus is on the alternative basis from the nonlinear moving average for local (perturbation) methods and we proceed accordingly.

[8] Our software add-on forces $N=0$ to align with Dynare (Adjemian et al., 2011). Thus in practice, the economist using Dynare must incorporate any exogenous serial correlation by including u_t in the vector y_t. This choice is not made in the exposition here as the admissibility of serial correlation in the exogenous driving force brings our first order derivation in line with earlier moving average approaches for linear models, e.g., Taylor (1986).

[9] The notation $\varepsilon_t^{\otimes[n]}$ represents Kronecker powers, $\varepsilon_t^{\otimes[n]}$ is the nth fold Kronecker product of ε_t with itself: $\underbrace{\varepsilon_t \otimes \varepsilon_t \ldots \otimes \varepsilon_t}_{n \text{ times}}$. For simulations, of course, more specific decisions regarding the distribution of the exogenous processes will have to be made. Kim et al. (2008, p. 3402) emphasize that distributional assumptions like these are not entirely local assumptions. Dynare (Adjemian et al., 2011) assumes normality of the underlying shocks.