Demand for cash with intra-period endogenous consumption

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ABSTRACT

We extend the literature on the demand for money by relaxing the assumption of a constant rate of consumption. Although total consumption is still fixed over the period, agents can choose more than one rate of consumption and cash depletion in the period to minimize the cost of money management. Consistent with empirical evidence, we find that agents do not smooth intra-period consumption. Instead, their rate of consumption will be positively related to their cash position. This positive correlation depends on the volatility of the consumption process.

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1. Introduction

To date, the literature on the demand for money has taken consumption as exogenous. Given the consumption process, which can be deterministic or stochastic, agents choose how much cash to withdraw at the start of a cycle. Cash holdings decline between withdrawals in line with the consumption process. Examples of this approach include the original contributions of Baumol (1952) and Tobin (1956) as well as the more recent work of Frenkel and Jovanovic (1980), Bar-Ilan (1990), Bar-Ilan et al. (2004), Baccarin (2009), and Alvarez and Lippi (2009, 2010).

The assumption of exogenous consumption actually involves two separate assumptions. First, total cash consumption expenditures over the period, and consequently total cash depleted over the period, are given. Second, the mean rate of cash consumption and cash depletion throughout the period is fixed.

As in the standard model of money demand, we retain the assumption of a given total cash consumption over the period and let it be determined by intertemporal considerations not modeled explicitly. However, we extend the literature on money demand by relaxing the assumption of a constant rate of cash consumption during the period. Instead, agents choose more than one mean rate of cash consumption that can vary with their cash position. This extension permits the decisions about within-period cash holdings and consumption rates to be made jointly.

Typically in the money demand literature, the consumption path over a period is characterized in continuous time as a Brownian motion (BM).[1] The parameters of the BM, namely the (negative) rate of drift and the instantaneous standard deviation of the process, are exogenous and fully characterize the path of consumption paid with cash and, correspondingly,

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  [1] In Bar-Ilan et al. (2004) and Alvarez and Lippi (2010), the underlying process is a combination of Brownian motion and Compound Poisson. This process is still exogenous and not under the control of agents.

the path of cash depletion.[2] Given the fixed parameters of the BM, as well as the opportunity cost of holding cash and the cost of restocking depleted cash balances, agents choose the optimal size of a cash withdrawal that minimizes total cost.

In our model, cash holdings and consumption are also characterized by a BM, but agents can select the drift. The model is therefore a drift control model, where the drift is the mean rate of consumption and cash depletion.[3] For simplicity, we initially give agents just two opportunities to optimize their consumption rate during the period, first when they make their initial cash withdrawal and again when cash balances are reduced by half. We then allow agents to optimize over their rate of consumption multiple times during a period.[4]

At the start of a typical period, agents choose the optimal cash withdrawal, M^*, the optimal rate of consumption for as long as money holdings do not reach $M^*/2$, and the optimal rate of consumption afterwards. When cash balances hit their lower bound, assumed here to be zero, a cash withdrawal of size M^* occurs and a new cycle starts. Agents therefore choose M^* and two rates of cash consumption in order to minimize the present value costs of withdrawing and holding money, subject to the constraint on total cash consumption over the period.

Numerical results for the model solution show unambiguously that cash consumption depends positively on cash holdings. The rate of cash consumption immediately after a cash withdrawal is greater than the consumption rate chosen at a lower cash level. Consequently, when agents have the opportunity to choose their rates of consumption over the period, they do not smooth consumption but prefer to consume at a higher rate initially.

The degree of consumption smoothing within a period depends on the amount of volatility in the consumption process. When volatility is either very high or very low, agents adopt a rapid rate of consumption immediately after a cash withdrawal and then slow down that rate later in the period. When volatility is large, the cash position can be high for a substantial period of time. To contain the high holding cost, agents choose a high rate of consumption at the start of the period. Low volatility allows consumers to make large purchases immediately upon a cash withdrawal because their cash consumption is predictable and they will not run out of cash prematurely. Consequently, the intra-period consumption path deviates substantially from consumption smoothing for high or low levels of volatility. For moderate volatility, agents get much closer to consumption smoothing within the period.

Our results appear consistent with some recent empirical work. For example, there are survey data on the demand for cash that indicate that cash consumption increases with the amount of cash held. Alvarez et al. (2010) uncovered evidence of simultaneous large cash withdrawals and large cash expenditures using a data set of rural Thai households. A Bank of Canada survey found that higher cash holdings led to a higher probability of paying with cash. (Arango et al., 2011). In addition, point estimates from money demand regressions generally reveal the income elasticity of cash to be less than 0.5, the value implied by the "square-root formula" derived in Baumol (1952). Boeschoten (1992) suggests this lower income elasticity may reflect a deviation from the implicit assumption in the standard money demand model of equally spaced expenditures between cash withdrawals. Instead, "households spend a large part of money relatively soon after its acquisition" (p. 61).

Similarly, survey data on American household consumption indicate that the path of within-the-month consumption may not be well represented by a single drift rate over the period. This is especially true for poor consumers who tend to finance more of their purchases with cash. For example, Stephens (2003) found that in the first few days following receipt of a Social Security check there is an increase in the amount of spending relative to the day before the check arrives. For poorer households, where Social Security represents a more significant portion of income, the spending increase at the beginning of the month is more pronounced. Mastrobuoni and Weinberg (2009) found evidence that Social Security recipients without savings do not smooth consumption over the month. Instead, these individuals consume 25 percent fewer calories the week before receiving Social Security checks relative to the week afterward. Of course, this behavior could also be explained by borrowing constraints rather than cash constraints. Alternatively, hyperbolic discounting could also generate non-smooth consumption that decreases within the month.

To emphasize the importance of cash consumption in economic activity, we conclude this section by highlighting the dominant role of cash in household transactions implied by European survey and diary data.[5] For example, Mooslechner et al. (2006) found that cash payments accounted for 86 percent of all direct payment transactions by Austrian households in 2005 and for 70 percent of total payment value. A Deutsche Bundesbank (2009) survey found that cash accounted for 82 percent of all direct payment transactions by German consumers in 2008 and for 58 percent in terms of value. Attanasio et al. (2002) found that currency is very important in the Italian payment system. Further, cash used by these European households for transactions was only a small part of total cash in circulation. The rest was hoarded, used in the shadow economy or held abroad.[6] Understanding more fully the management of cash holdings is therefore an important goal.

The rest of the paper is organized as follows. In Section 2 we present the model and its solution when agents have only two opportunities to optimize their rates of cash consumption and cash depletion over the period. Detailed derivations,

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[2] When applied to firms, the BM process represents the firms' net revenue in cash.

[3] Bar-Ilan et al. (2007) illustrate another type of drift control model.

[4] Limiting the number of times agents observe their cash position and adjust rates of consumption is consistent with the literature on optimal inattention to the stock market (e.g. Abel et al. (2007)).

[5] Our model implicitly assumes a cash-in-advance constraint or the existence of cash goods as in Lucas and Stokey (1983). Household diary data indicate that cash is only one of several methods of payment. Unlike Europe, in the U.S., households use debit cards more frequently than either cash, credit cards or checks. (Federal Reserve Bank of Boston, 2010).

[6] Schneider et al. (2010) estimated that the average value of the shadow – or cash – economy was 34.5% of official GDP for 162 countries between 1999 and 2006/7. The cash economy was 38.7% of official GDP for a group of 98 developing countries and 18.7% for a group of 25 high-income countries.